

PRE-CALCULUS



Topic: _____ Name: _____

Main Ideas/Questions Notes/Examples

Angles in STANDARD FORM

- An angle on the coordinate plane has its vertex at the origin (0,0).
- The ray on the x-axis is called the initial side.
- The other ray is called the terminal side.
- Counterclockwise rotation is positive.
- Clockwise rotations are negative.

DEGREES

- The most common unit for measuring angles is degrees.
- A degree is equivalent to $\frac{1}{360}$ of a full rotation.

Sketch an angle in standard form.

1. 65°

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CIRCULAR MOTION

Arc length can be used to measure the distance an object moves. Suppose an object moves along a circular path with radius r and central angle θ (in radians). The arc length s is given by the formula:

$$s = r\theta$$

Linear Speed Formula:

$$v = r\omega$$

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TRIGONOMETRIC FUNCTIONS

Trigonometric functions are defined by the trigonometric ratios of a right triangle. The sine function is defined by the ratio of the opposite side to the hypotenuse. The cosine function is defined by the ratio of the adjacent side to the hypotenuse. The tangent function is defined by the ratio of the opposite side to the adjacent side.

The six trigonometric functions are:

Function	Definition
Sine	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
Cosine	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
Cosecant	$\csc \theta = \frac{1}{\sin \theta}$
Secant	$\sec \theta = \frac{1}{\cos \theta}$
Cotangent	$\cot \theta = \frac{1}{\tan \theta}$

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REFERENCE ANGLES

For an angle θ in standard position, the reference angle is the acute angle between the terminal side and the x-axis. Sketch each angle and find its reference angle.

1. 150°

2. 315°

4. $\frac{5\pi}{3}$

5. $\frac{7\pi}{6}$

Use reference angles to find the values of the trigonometric functions.

Recall the signs of the functions in each quadrant:

Quadrant	Sine	Cosine	Tangent
I	+	+	+
II	+	-	-
III	-	-	+
IV	-	+	-

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TRANSLATIONS of Trigonometric Functions

Like other functions, trigonometric functions can be translated. Given the functions $f(x) = a \sin(bx + c)$ and $g(x) = a \cos(bx + c)$, the amplitude is $|a|$, the period is $\frac{2\pi}{|b|}$, and k represents the vertical shift.

Example: $f(x) = \frac{3}{2} \sin x + 3$

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RECIPROCAL FUNCTIONS

The reciprocal functions are the cosecant, secant, and cotangent functions. Their graphs are shown below.

COSECANT $f(x) = \csc x$

SECANT $f(x) = \sec x$

COTANGENT $f(x) = \cot x$

NOTES • HOMEWORK • QUIZZES • TEST

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Unit 5 - Trigonometric Functions: Sample Unit Outline

	TOPIC	HOMEWORK
DAY 1	Standard Form of an Angle; Degrees vs. Radians, Coterminal Angles; Degree-Minute-Second Form	HW #1
DAY 2	Arc Length and Area of Sectors	HW #2
DAY 3	Circular Motion: Linear and Angular Speed	HW #3
DAY 4	Quiz 5-1	None
DAY 5	Six Trigonometric Functions (sin, cos, tan, csc, sec, cot); Trigonometric Values in Special Right Triangles	HW #4
DAY 6	Finding Side & Angle Measures; Solving Right Triangles: Applications	HW #5
DAY 7	Trigonometric Functions for Any Angle; Reference Angles & The Unit Circle	HW #6
DAY 8	Quiz 5-2	None
DAY 9	Law of Sines	HW #7
DAY 10	Law of Cosines	HW #8
DAY 11	Mixed Law of Sines and Law of Cosines; Area of Triangles	HW #9
DAY 12	Quiz 5-3	None
DAY 13	Graphing Sine, Cosine, and Tangent Functions	HW #10
DAY 14	Translating Sine, Cosine, and Tangent Functions (horizontal/vertical shifts and vertical shifts)	HW #11
DAY 15	Graphing Reciprocal Functions	HW #12
DAY 16	Graphing Trigonometric Functions Review	HW #13
DAY 17	Quiz 5-4	None
DAY 18	Inverse Trigonometric Functions	HW #14
DAY 19	Compositions of Trigonometric Functions	HW #15
DAY 20	Unit 5 Review	Study For Test
DAY 21	UNIT 5 TEST	None
ADDITIONAL RESOURCES: Video Links & Google Slides Suggested Activities		

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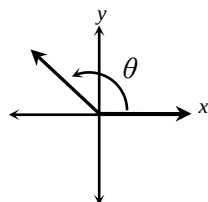
Topic:

Class:

Main Ideas/Questions

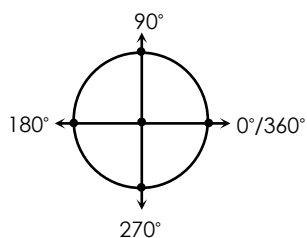
Notes/Examples

Angles in STANDARD FORM



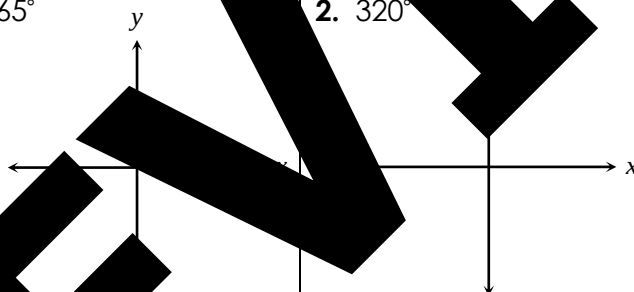
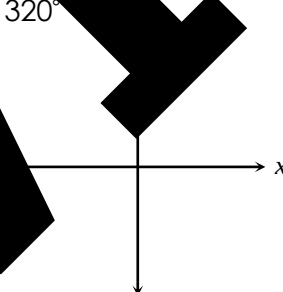
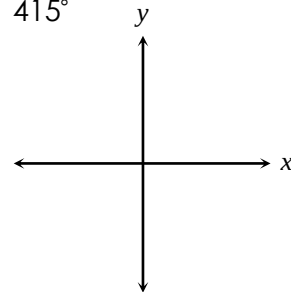
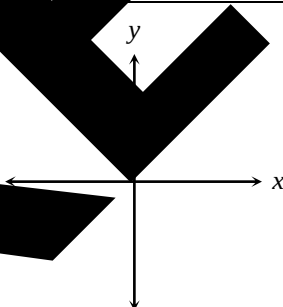
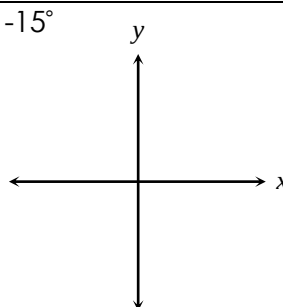
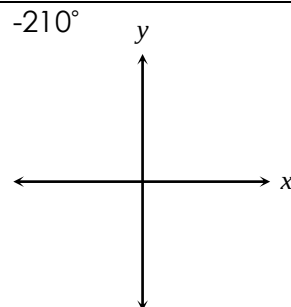
- An angle on the coordinate plane is in standard form when the vertex is on the origin and one ray lies on the positive x-axis.
- The ray on the x-axis is called the terminal side.
- The other ray is called the initial side.
- Counterclockwise rotations result in positive angle measures.
- Clockwise rotations result in negative angle measures.

DEGREES

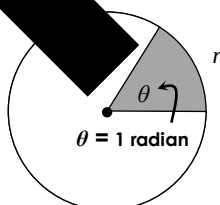


- The most common unit of measurement is the degree, (°).
- A degree is equivalent to $\frac{1}{360}$ of a full rotation.

Sketch an angle in standard form for the given measure.

1. 65° 2. 320° 3. 415° 4. 110° 5. -15° 6. -210° 

RADIANS



A **radian** is a unit of angle measure based on arc length. One radian is defined as the measure of the angle formed when the radius is equivalent to the length of the intercepted arc. Recall that the circumference of a circle is $2\pi r$, therefore:

$$360^\circ = \frac{2\pi r}{r}; \quad 180^\circ = \frac{\pi r}{r}$$

Converting Degrees → Radians

$$\text{Radians} = \text{Degrees} \cdot \left(\frac{\pi \text{ radians}}{180} \right)$$

Converting Radians → Degrees

$$\text{Degrees} = \text{Radians} \cdot \left(\frac{180}{\pi \text{ radians}} \right)$$

Converting DEGREES TORADIANS	Convert each angle measure to radians.		
	7. 60°	8. 160°	9. 330°
	10. -45°	11. -230°	12. -75°
Converting RADIANS TODEGREES	Convert each angle measure to degrees.		
	13. $\frac{5\pi}{18}$	14. $\frac{11\pi}{9}$	15. $\frac{7\pi}{3}$
	16. $-\frac{\pi}{6}$	17. $-\frac{5\pi}{4}$	18. $-\frac{3\pi}{4}$
COTERMINAL ANGLES	Angles in standard position with the same terminal side are coterminal angles. Determine if the two angles are coterminal.		
	19. 0° and -55°	20. 30° and -170°	
	21. $\frac{\pi}{4}$ and $\frac{9\pi}{4}$	22. $\frac{3\pi}{2}$ and $-\frac{5\pi}{2}$	
	For each given angle, find two coterminal angles for each given angle, one positive and one negative.		
	23. 150°	24. 265°	25. -25°
	26. $\frac{\pi}{12}$	27. $\frac{7\pi}{6}$	28. $-\frac{11\pi}{18}$

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Main Ideas/Questions	Notes/Examples							
DEGREE-MINUTE-SECOND FORM (DMS)	Some angles have decimal degrees (for example, 26.5°). Angles with decimals can be expressed using degree-minute-second form (DMS) in which the degrees are subdivided into minutes and seconds as follows: one degree = _____ minutes; one minute = _____ seconds *one degree = _____ seconds Symbols for degree, minutes, and seconds <table><tr><td>DEGREE</td><td>MINUTE</td><td>SECOND</td></tr><tr><td></td><td></td><td></td></tr></table>		DEGREE	MINUTE	SECOND			
	DEGREE	MINUTE	SECOND					
Converting DECIMAL DEGREES TO DMS	Follow the example below to convert 31.875° to degree-minute-second form. 31.875° = ° ' " ° = ° ' " Therefore, ° =							
	Convert each angle measure in DMS form.							
	1. 150.2°							
	3. 89.915°	4. 340.375°						
	5. 48.526°	6. 273.771°						

	7. 105.394°	8. 318.812°
Converting DMSTO DECIMAL DEGREES	Follow the example below to convert $73^\circ 9' 50''$ to decimal degrees.	
	$73^\circ 9' 50'' \rightarrow$ / / =	
	Therefore, $73^\circ 9' 50'' =$ $=$	
	Write each angle measure in decimal degree form.	
	9. $37^\circ 36'$	10. $3^\circ 12' 18''$
	11. $12^\circ 2' 18''$	12. $121^\circ 4' 45''$
	13. $22^\circ 20''$	14. $8^\circ 34' 16''$
	15. $50^\circ 14' 56''$	16. $253^\circ 46' 5''$

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Unit 5: Trigonometric Functions

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Homework 1: Angle Measures

**** This is a 2-page document! ******Directions:** Convert each angle measure to radians.

1. 140°

2. -75°

3. 190°

4. 312°

5. -65°

6. -225°

Directions: Convert each angle measure to degrees.

7. $\frac{3\pi}{5}$

8. $-\frac{7\pi}{18}$

9. $\frac{10\pi}{9}$

10. $-\frac{9\pi}{5}$

11. $\frac{11\pi}{12}$

12. $\frac{15\pi}{4}$

Directions: Find two coterminal angles for each given angle, one positive and one negative.

13. 195°

14. 319°

15. -64°

16. $\frac{3\pi}{5}$

17. $\frac{11\pi}{7}$

18. $\frac{19\pi}{6}$

Directions: Write each angle measure in Degree-Minute-Second (DMS) form.

19. 36.875°

20. 117.590°

21. -238.145°

22. 64.901°

23. -323.588°

24. -136°

Directions: Write each angle measure in Degree-Minute-Second (DMS) form.

25. $54^\circ 26' 18''$

26. $-134^\circ 37' 20''$

27. $-98^\circ 12' 14''$

28. $289^\circ 5' 30''$

29. $-129^\circ 14' 14''$

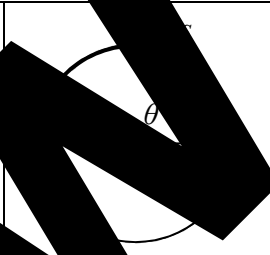
30. $100^\circ 45' 6''$

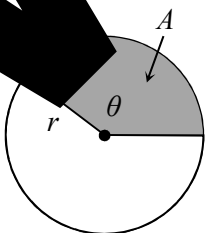
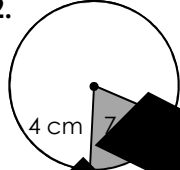
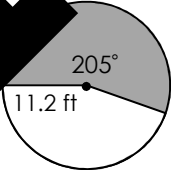
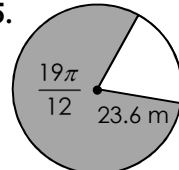
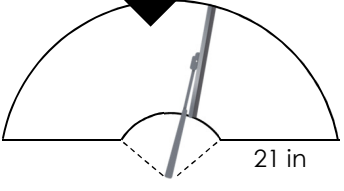
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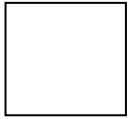
Class:

Main Ideas/Questions	Notes/Examples	
ARC LENGTH Examples Convert to radians	If θ is a central angle in a circle with radius r, then the length of the intercepted arc, s, is given by: (where θ is measured in radians)	
	Find the length of the intercepted arc given the angle measure and radius. Give your answer in terms of π and round to the nearest tenth.	
	1. $\frac{5\pi}{6}$; $r = 3$ ft	2. $\frac{\pi}{4}$; $r = 0.5$ in
	3. $\frac{3\pi}{2}$; $r = 0.5$ yd	4. $\frac{\pi}{3}$; $r = 6.2$ yd
	5. 120°; $r = 4$ m	6. 330°; $r = 10.5$ mm
	7. 120°; $r = 1.8$ ft	8. 135°; $r = 25$ km
	9. The central angle θ in a circle of radius 6 meters has an intercepted arc length of 10 meters. Find the measure of θ in radians and in degrees.	

	10. A merry go round rotates 2808° per ride. How far would a rider seated 8 feet from the center of the merry go round travel during the ride?
	11. Cincinnati, Ohio is directly north of Atlanta, Georgia. Cincinnati has a latitude of 39.1° N and Atlanta has a latitude of 33.7° N. If the earth has a radius of 3,960 miles, how far apart are these cities?
AREA OF SECTORS	The area A of a sector of a circle with radius r and central angle θ is given by: (where θ is measured in radians) 
<i>Examples</i>	Find the area of each sector. 12.  13.   15. 
	16. The area of a sector of a circle with a central angle of 240° is 134 ft^2. Find the radius of the circle.
	17. The windshield wiper arm to the left is 32 inches long. If the wiper sweeps through an angle of 125°, find the area swept by the blade.

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Homework 2: Arc Lengths & Area of Sectors

**** This is a 2-page document! ******Directions:** Find the length of each intercepted arc given the angle measure and radius. Give your answer in terms of π and rounded to the nearest tenth.

1. $\frac{3\pi}{4}; r = 10$ mm

2. $\frac{8\pi}{7}; r = 17.5$ yd

3. $\frac{\pi}{6}; r = 2$ mi

4. $\frac{5\pi}{8}; r =$

5. $320^\circ; r = 0.9$ cm

6. $100^\circ; r =$

7. $240^\circ; r = 6.6$ km

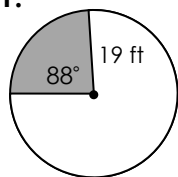
8. $45^\circ; r = 2.2$ m

9. An intercepted arc has a length of 19 yards. If the radius is 4 yards, find the measure of the central angle in radians and degrees.

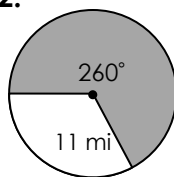
10. The tip of an hour hand on a clock is 5.5 inches from the center. How far will the hour hand travel over three full days?

Directions: Find the area of each sector.

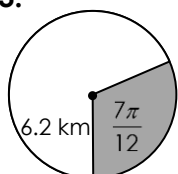
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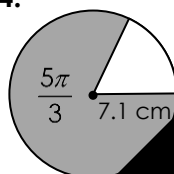
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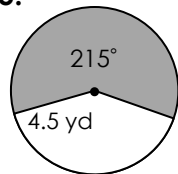
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14.



15.



16.

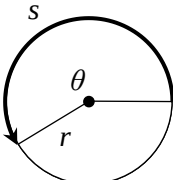


17. The sector of a circle has a central angle of 108° and an area of 70.7 square meters. Find the radius of the circle.

18. A circle with a radius of 12 yd contains a sector with an area of 609 yd². Find the measure of the central angle of the sector in both radians and degrees.

19. A clock is equally divided into 12 sections. If the clock reads 8:00 and has a diameter of 12 inches, find the area of the smaller sector formed by the minute and hour hands.

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CIRCULAR MOTION 	Arc length can be used to analyze circular motion. Suppose an object moves along a circular path with radius r.	
	The rate at which the object moves along the path is called its linear speed, v. Linear Speed Formula:	The rate at which the central angle is changing along with the arc length is called angular speed, ω. Angular Speed Formula:
Examples Recall: Each rotation (or revolution) has a circumference of $2\pi r$ and a central angle of 2π radians.	Use for questions 1-2: A bicycle tire with a radius of 14 inches rotates at a rate of 125 revolutions per minute (rpm).	
	1. Find the linear speed of the tire in inches per minute.	2. Find the angular speed of the tire in radians per minute.
	For questions 3-4: A CD with a diameter of 120 millimeters rotates a rate of 400 rpm.	
	3. Find the linear speed of the CD in millimeters per minute.	4. Find the angular speed of the CD in radians per minute.
	5. A 16-inch diameter tire on a car is making 500 revolutions per minute. Find the linear speed of the tire in miles per hour.	

6. A circular saw blade with a diameter of 9 inches rotates at 2800 revolutions per minute. Find the angular speed of the blade in radians per second.

7. A windmill has blades that are 14 feet long. If the windmill is rotating at 12 revolutions per second, find the linear speed of the tip of the blades in miles per hour.

Using
ANGULAR SPEED
to find
LINEAR SPEED

The linear speed, v , can be found as follows:

$$\boxed{} \boxed{} = \boxed{} = \boxed{}$$

Therefore, you can use the angular speed, ω , to find the linear speed, v .

Example: A windmill with 25-inch blades rotates at 40 rpm. Find the linear speed of the tip of the blades in feet per second.

9. Ryan is riding a bicycle whose wheels are 28 inches in diameter. If the wheels rotate at 130 rpm, find the linear speed in miles per hour in which he is traveling.



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Homework 3: Linear & Angular Speeds

**** This is a 2-page document! ****

Use for questions 1 and 2: An SUV tire with a radius of 12.5 inches rotates at a rate of 545 revolutions per minute.

- | | |
|--|--|
| 1. Find the linear speed of the tire in inches per minute. | 2. Find the angular speed of the tire in radians per minute. |
|--|--|

Use for questions 3 and 4: The fan on a pool heater has a diameter of 107 centimeters and rotates at a rate of 212 revolutions per minute.

- | | |
|--|---|
| 3. Find the linear speed of the fan in centimeters per minute. | 4. Find the angular speed of the fan in radians per minute. |
|--|---|

5. A skateboard wheel with a 56-centimeter diameter makes 430 revolutions per minute. Find the linear speed of the wheel in meters per second.

6. A bicycle tire with a 12-inch radius is making 175 revolutions per minute. Find the linear speed in miles per hour.

7. A record with a diameter of 12 inches rotates at 33 revolutions per minute. Find the angular speed of the record in radians per second.

8. An oscillating fan has blades that measure 8 inches long. If the fan rotates at a speed of 6 revolutions per second, find the linear speed of the ends of the blades in inches per hour.

9. The wheels on Darlene's car have an 11-inch radius. If the wheels are rotating at a rate of 378 rpm, find the linear speed in miles per hour in which she is traveling.

10. A DVD with a radius of 6 centimeters rotates at a speed of 243 revolutions per minute. Find the linear speed in centimeters per second.

11. A lawnmower blade has a diameter of 30 inches. If the blade rotates at a speed of 126 revolutions per minute, find the linear speed of the tips of the blades in feet per second.

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Unit 5: Trigonometric Functions

Quiz 5-1: Angle Measures, Arc Lengths, Area of Sectors, & Circular Motion

Write the following angle measures in radians.

1. 165°

2. -300°

Write the following angle measures in degrees.

3. $-\frac{\pi}{10}$

4. $\frac{7\pi}{6}$

Give one positive and one negative coterminal angle for each given angle.

5. 135°

6. $-\frac{\pi}{4}$

Write the following angle measures in degree-minute-second (DMS) form.

7. 42.25°

8. -210.615°

Write the following angle measures in decimal degree form.

9. $164^\circ 39'$

10. $8^\circ 15' 54''$

Find the length of the intercepted arc given the central angle and radius of the circle. Round your answer to the nearest tenth.

11. $\theta = \frac{\pi}{5}$; $r = 9$ cm

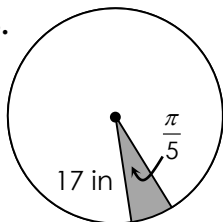
12. $\theta = 345^\circ$; $r = 2.5$ ft

11. _____

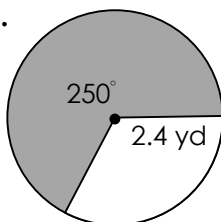
12. _____

Find the area of each sector. Round your answer to the nearest tenth.

13.



14.



13. _____

14. _____

Use your knowledge of arc lengths, area of sectors, and circular motion to solve each problem.

15. Lincoln, Nebraska lies directly north of Dallas, Texas. Lincoln is at a latitude of 40.5° N and Dallas is at a latitude of 32.8° N. Assuming the radius of the earth is 3,960 miles, how far apart are these cities?

15. _____

17. _____

18. _____

16. A motion detector can detect movement up to 25 feet away through an angle of 105° . What is the area of the motion detector monitor?

17. The diameter of each tire on a vehicle is 32 inches. If the tires are moving at a rate of 800 revolutions per minute, find the linear speed of the vehicle in miles per hour.

18. A drum in a washing machine spins at 1,200 revolutions per minute. If the diameter of the drum is 76 centimeters, find the angular speed of the drum in radians per second.

Use the given trigonometric value to find the remaining function values.

5. $\tan \theta = \frac{7}{24}$

6. $\csc \theta = \frac{37}{12}$

$\sin \theta =$

$\csc \theta =$

$\sin \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

$\tan \theta =$

$\cot \theta =$

7. $\cos \theta = \frac{5}{9}$

$\sec \theta =$

$\sin \theta =$

$\csc \theta =$

$\sin \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

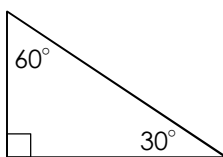
$\tan \theta =$

$\cot \theta =$

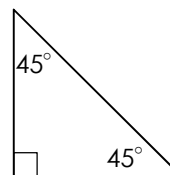
TRIG VALUES SPECIAL ANGLES

The angles of 30° , 45° , and 60° are used frequently in trigonometry. You will use your knowledge of the side relationships in special right triangles to find the values of the trigonometric ratios.

30°-60°-90° Triangle



45°-45°-90° Triangle



θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
30°						
45°						
60°						

Name: _____

Unit 5: Trigonometric Functions

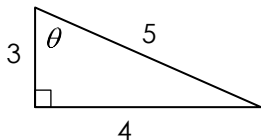
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Homework 4: Trigonometric Functions

**** This is a 2-page document! ****

Directions: Find the exact values of the trigonometric functions of θ .

1.



$\sin \theta =$

$\csc \theta =$

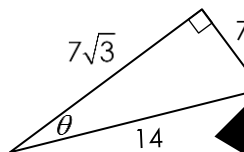
$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

2.



$\sin \theta =$

$\csc \theta =$

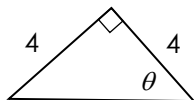
$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

3.



$\sin \theta =$

$\csc \theta =$

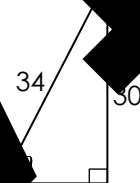
$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

4.



$\sin \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

5.



$\sin \theta =$

$\csc \theta =$

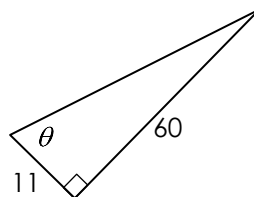
$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

6.



$\sin \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

Directions: Use the given trigonometric function value of θ to find the exact values of the remaining functions.

7. $\cos \theta = \frac{60}{61}$

8. $\csc \theta = 7$

$\sin \theta =$

$\csc \theta =$

$\sin \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

$\tan \theta =$

$\cot \theta =$

9. $\sin \theta = \frac{\sqrt{2}}{2}$

10. $\tan \theta = \frac{1}{3}$

$\sin \theta =$

$\csc \theta =$

$\tan \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

$\tan \theta =$

$\sec \theta =$

$\tan \theta =$

$\cot \theta =$

$\tan \theta =$

$\cot \theta =$

11. $\sec \theta = \frac{\sqrt{35}}{5}$

12. $\cot \theta = \frac{\sqrt{11}}{7}$

$\csc \theta =$

$\sin \theta =$

$\csc \theta =$

$\sec \theta =$

$\cos \theta =$

$\sec \theta =$

$\cot \theta =$

$\tan \theta =$

$\cot \theta =$

Directions: Give the exact value of each function.

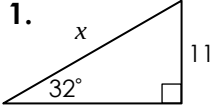
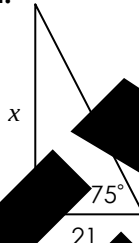
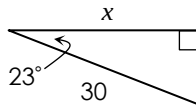
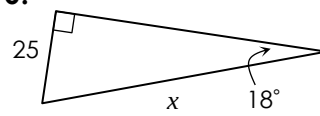
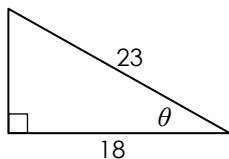
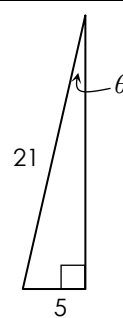
13. $\cos 60^\circ$

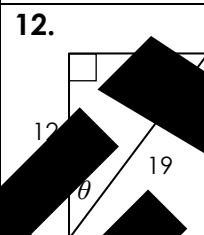
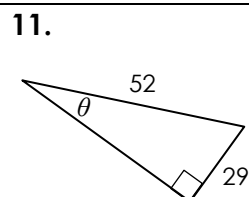
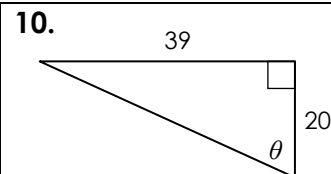
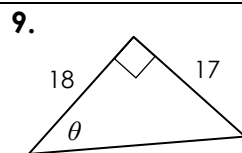
14. $\tan 45^\circ$

15. $\csc 45^\circ$

16. $\cot 30^\circ$

Name:	Date:
Topic:	Class:

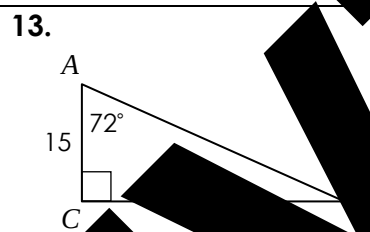
Main Ideas/Questions	Notes/Examples		
<i>Finding</i> SIDE LENGTHS	Find the value of x . Round to the nearest tenth.		
	1. 	2. 	
	3. 		
	5. 	6. 	
<i>Finding</i> ANGLE MEASURES	Given the trigonometric value of an angle, use the corresponding inverse trigonometric function to find the measure of the angle.		
	Inverse Sine	Inverse Cosine	Inverse Tangent
	If $\sin \theta = x$, then	If $\cos \theta = x$, then	If $\tan \theta = x$, then
$\sin^{-1} x$ is read as the inverse sine of x.	Find the measure of θ . Round to the nearest tenth of a degree.		
	7. 	8. 	



Solving RIGHT TRIANGLES

Solving a right triangle means to find all side and angle measures.

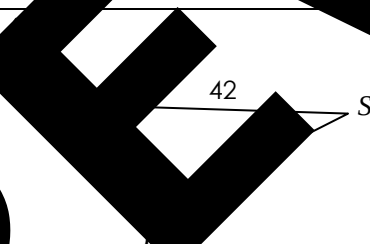
Solve each triangle. Round measures to the nearest tenth.



$BC =$

$AB =$

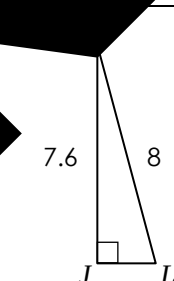
$m\angle B =$



$RT =$

$ST =$

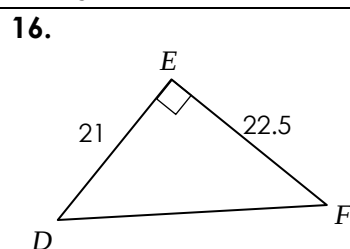
$m\angle R =$



$JL =$

$m\angle K =$

$m\angle L =$



$DF =$

$m\angle D =$

$m\angle F =$

Name:

Date:

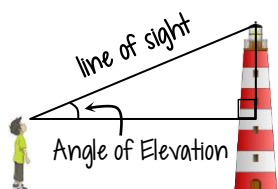
Topic:

Class:

Main Ideas/Questions

Notes/Examples

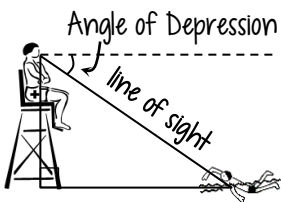
ANGLE OF ELEVATION



Round all answers to the nearest tenth when necessary.

1. A fire started at a campsite located 8 miles from the base of a mountain. If the angle of elevation from the fire to the top of the mountain is 14° , how high is the mountain?
2. A man is standing 85 meters from the base of a tower. The angle of elevation from the base of the tower to the top of the tower is 20° . How high is the tower?
3. A 32-foot tall ladder leans against a vertical wall. If the top of the ladder reaches a point 25 feet up the wall, how far is the bottom of the ladder from the ground?
4. A kite is flying 60 feet above the ground. The angle of elevation from where her hand is holding the string to the top of the kite is $48^\circ 18'$. If her hand is 5 feet from the ground, find the length of the string.
5. A ramp is being built to access a door that is 1.5 feet above the ground. Find the length of the ramp if the angle of elevation must be 8° .
6. Sara is standing 35 feet from the base of a rock wall watching her two sons, Max and Jon, climb the wall. The angle of elevation is 38° to Max and 27° to Jon. How many feet above Jon is Max?

ANGLE OF DEPRESSION



7. Building A is 464 feet tall and Building B is 321 feet tall. Dale is standing between the buildings. The angle of elevation from the point on the ground where Dale is standing is 75° to the top of Building A and 48° to top of Building B. How far apart are the buildings?

8. From the top of a lighthouse 150 feet above sea level, the angle of depression to a boat at sea is 28° . What is the horizontal distance from the boat to the base of the lighthouse?

9. The angle of depression from a helicopter to a boat at sea is $34^\circ 8''$. If the horizontal distance from the helicopter to the boat is 1,200 feet, find the altitude of the helicopter.

10. Henry is standing on a tree 50 feet above the ground. He is looking down at a deer. If the deer is 30 feet from the base of the tree, find the angle of depression from Henry to the deer.

11. A wire is attached from the top of a post to a point on the ground 24 feet from the base of the post. If the angle of depression from the top of the post to the point where the wire is attached to the ground is 61° , find the length of the wire.

12. A scuba diver spots a shipwreck at an angle of depression of 26° . The angle of elevation from the scuba diver up to his boat is 53° . If the shipwreck is located directly below his boat, and the horizontal distance between the scuba diver and the shipwreck is 120 feet, find the vertical distance between the shipwreck and the boat.

Name: _____

Unit 5: Trigonometric Functions

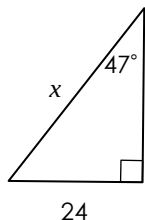
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Homework 5: Solving Right Triangles

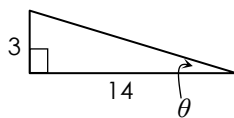
**** This is a 2-page document! ****

Directions: Find the missing side or angle measure. Round answers to the nearest tenth.

1.



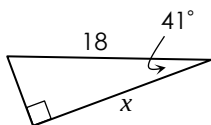
2.



3.

39

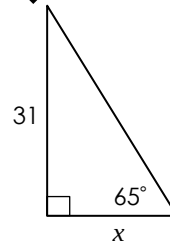
4.



5.

7

6.



Directions: Solve each triangle. Round all measures to the nearest tenth.

7.

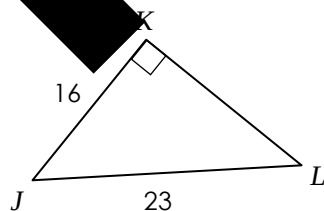


$AB =$

$AC =$

$m\angle C =$

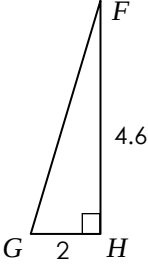
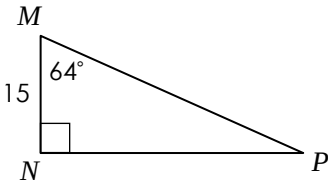
8.



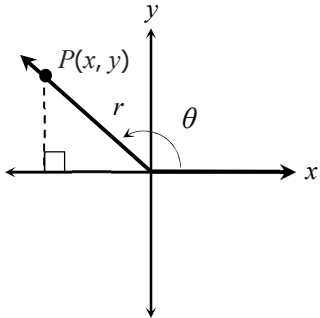
$KL =$

$m\angle J =$

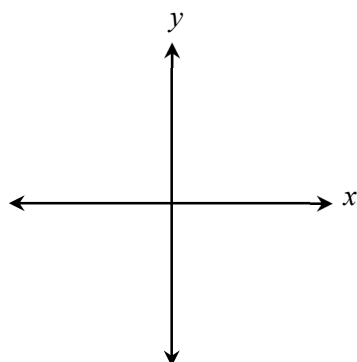
$m\angle L =$

9. 	$FG =$
10. 	$MP =$ $m\angle P =$
11. A 50-foot ladder is leaning against a vertical wall. If the base of the ladder is 45 feet from the base of the wall, find the angle the bottom of the ladder makes with the ground.	12. Joel is standing on the mountains. If the vertical distance from his platform to the next platform is 4 yds and the angle of depression is 20°, find the length of the line.
13. A string connects the corner of a square canopy to a point on the ground 15 feet from the corner. If the angle of elevation from the point on the ground to the corner of the canopy is 37°, find the length of the string.	14. At a hot air balloon festival, Erica spots a red balloon due east at an angle of elevation of 36°. She turns due west and sees a blue balloon at an angle of elevation of 51°. If altitude of the red balloon is 296 meters, the altitude of the blue balloon is 461 meters, and Erica is 1.6 meters tall, find the horizontal distance between the balloons.

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples		
TRIG FUNCTIONS for any angle 	In the previous lesson, you found trigonometric functions for positive acute angles. We will now find trigonometric functions for any angle. Let θ be an angle in standard form and $P(x, y)$ be a point on the terminal side of θ. The distance from P to the origin, r, can be found using the Pythagorean Theorem: $r = \sqrt{x^2 + y^2}$		
	$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
	$\csc \theta =$	$\sec \theta =$	$\cot \theta =$
	1. $P(9, -12)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ.		
	$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
	$\csc \theta =$	$\sec \theta =$	$\cot \theta =$
	2. $P(-5, 12)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ.		
	$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
	$\csc \theta =$	$\sec \theta =$	$\cot \theta =$
	$\sin \theta =$	$\cos \theta =$	$\tan \theta =$
	$\csc \theta =$	$\sec \theta =$	$\cot \theta =$

SIGNS OF TRIG FUNCTIONS



Depending on the location of the terminal side of the angle, trigonometric functions can be positive or negative. Using your knowledge of the trigonometric function values, identify the functions that are positive in each quadrant.

Quadrant I

Quadrant II

Quadrant III

Quadrant IV

Remember the phrase "All Students Take Calculus"

3. If $\cos \theta > 0$, which quadrant(s) could the terminal side of θ lie?

4. If $\tan \theta < 0$, which quadrant(s) could the terminal side of θ lie?

5. If $\sec \theta < 0$, which quadrant(s) could the terminal side of θ lie? If $\sin \theta > 0$ and $\cos \theta < 0$, which quadrant(s) could the terminal side of θ lie?

USING

TRIG VALUES

to find other values

Use the given information to find the exact values of the five remaining trigonometric functions of θ .

7. $\sin \theta = \frac{8}{17}$; $\tan \theta > 0$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\sec \theta =$

$\cot \theta =$

8. $\csc \theta = 4$; $\sec \theta < 0$

$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\sec \theta =$

9. $\csc \theta = -\frac{\sqrt{5}}{2}$; $\cos \theta < 0$

$\sin \theta =$

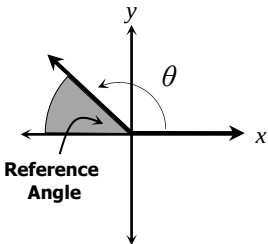
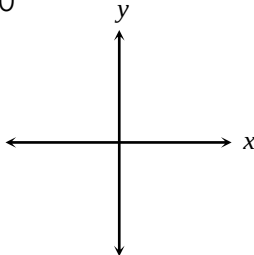
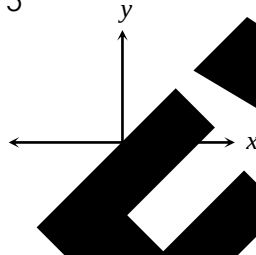
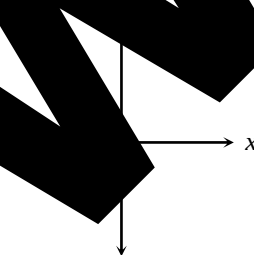
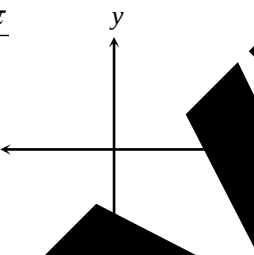
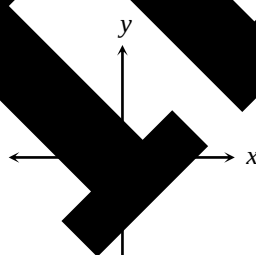
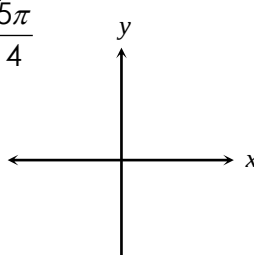
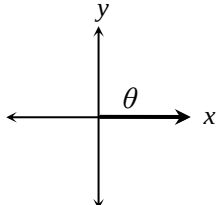
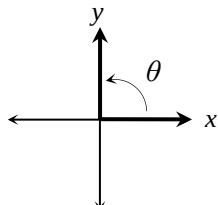
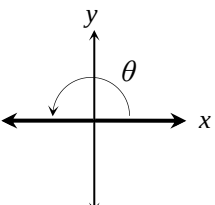
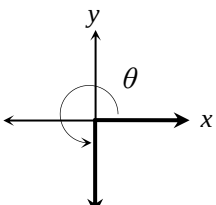
$\cos \theta =$

$\tan \theta =$

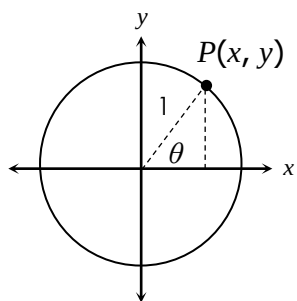
$\sec \theta =$

$\cot \theta =$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples			
REFERENCE ANGLES 	For an angle θ in standard form, the reference angle is the positive acute angle form by the terminal side and the x-axis. Sketch each angle, then find its reference angle.			
	1. 150° 	2. 315° 	3. -225° 	
	4. $\frac{5\pi}{3}$ 			
	Use reference angles and trigonometric function values for angles in special right triangles to find each trigonometric value.			
Recall the signs of the functions in each quadrant!	7. $\sec 135^\circ$	8. $\sec 225^\circ$		
	9. $\csc \frac{11\pi}{6}$	10. $\csc \left(-\frac{2\pi}{3}\right)$		
QUADRANTAL ANGLES	When the terminal side of an angle θ that is in standard position lies on the x- and y-axis, the angle is called a quadrantal angle .			
	$\theta = 0^\circ$ (or 0π) 	$\theta = 90^\circ$ (or $\frac{\pi}{2}$) 	$\theta = 180^\circ$ (or π) 	$\theta = 270^\circ$ (or $\frac{3\pi}{2}$) 

THE UNIT CIRCLE



A unit circle is a circle with a radius of 1 unit.
Because the value of r is 1 for each point $P(x, y)$ on the circle, the trigonometric functions of θ are defined as:

$$\sin \theta =$$

$$\cos \theta =$$

$$\tan \theta =$$

$$\csc \theta =$$

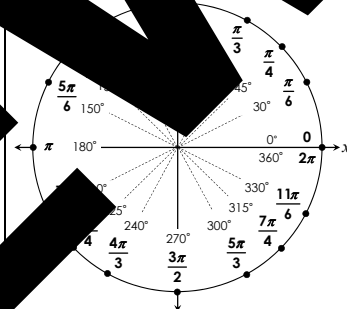
$$\sec \theta =$$

$$\cot \theta =$$

* The coordinates of P can be written as _____

16-POINT UNIT CIRCLE

The unit circle is frequently used to find the values of the trigonometric functions for the angles 0° through 360° (and their multiples) as well as the values of the trigonometric functions for the angles 0 through 2π (the quadrant angles).



THE UNIT CIRCLE & TRIG VALUES

Complete "The Unit Circle Reference" then use the circle to find the exact value of each trigonometric function.

11. $\sin 135^\circ$

12. $\sec 180^\circ$

13. $\cot (-300^\circ)$

15. $\tan 270^\circ$

16. $\cos (-150^\circ)$

17. $\cos \frac{3\pi}{4}$

18. $\tan \frac{2\pi}{3}$

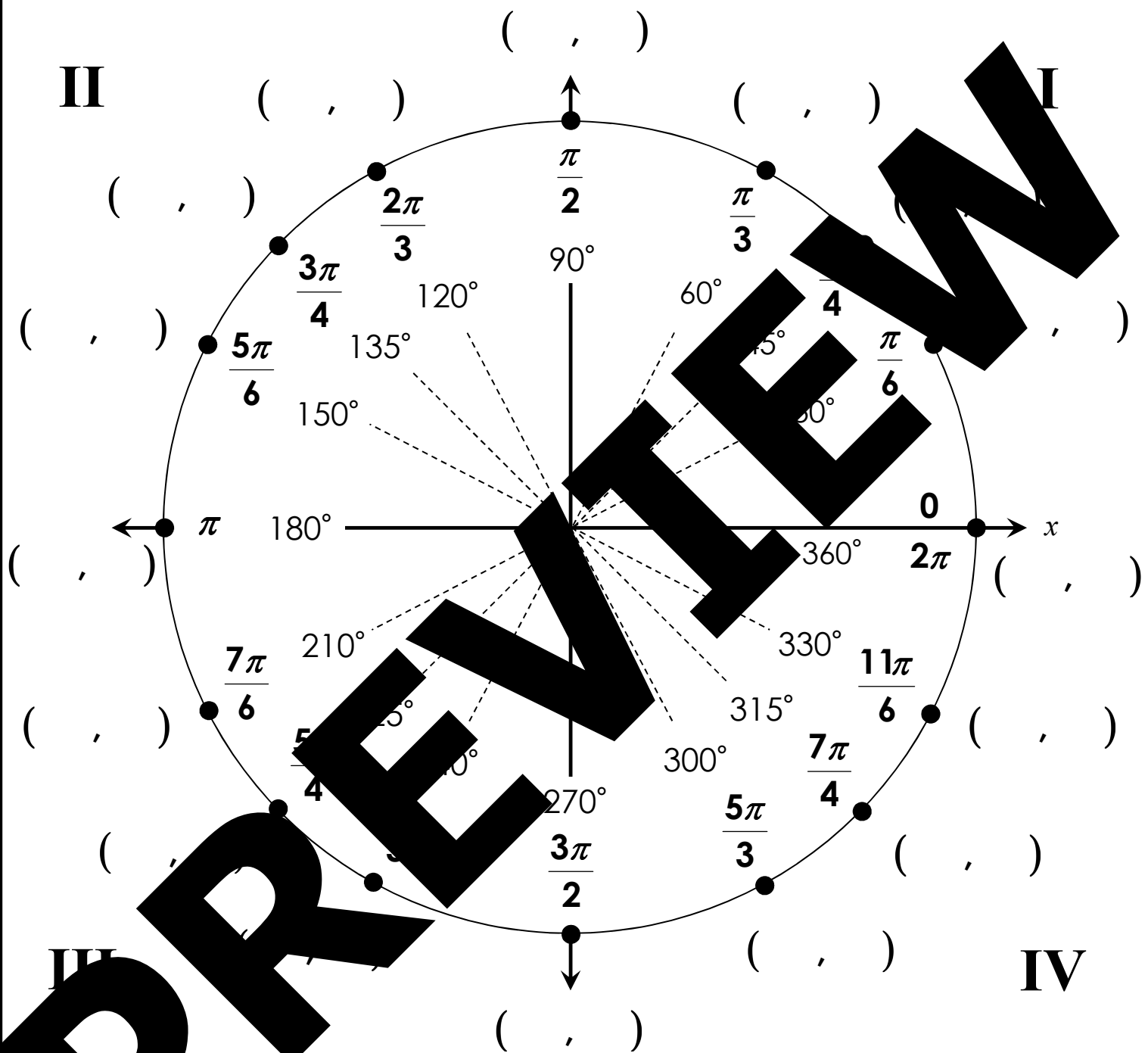
19. $\cot \frac{\pi}{2}$

20. $\sec \left(-\frac{\pi}{4} \right)$

21. $\tan \frac{5\pi}{4}$

22. $\csc \left(-\frac{5\pi}{6} \right)$

THE UNIT CIRCLE *Reference!*



Positive Trigonometric Values:

Quad I	
Quad II	
Quad III	
Quad IV	

Trigonometric Values:

$\sin \theta =$	$\csc \theta =$
$\cos \theta =$	$\sec \theta =$
$\tan \theta =$	$\cot \theta =$

Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 6: Trig Functions of Any Angle & The Unit Circle

**** This is a 2-page document! ******Directions:** Given a point on the terminal side of θ in standard form, find the exact values of the trigonometric functions of θ .

1. $P(-5, 12)$

$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\sec \theta =$

$\cot \theta =$

2. $P(-4, -10)$

$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\sec \theta =$

$\cot \theta =$

Directions: Use the given information to find the exact values of the five remaining trigonometric functions of θ .

3. $\tan \theta = -\frac{2}{3}; \sin \theta < 0$

$\sin \theta =$

$\cos \theta =$

$\csc \theta =$

$\sec \theta =$

$\cot \theta =$

4. $\csc \theta = -\sqrt{10}; \cot \theta > 0$

$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\cot \theta =$

Directions: Use reference angles and the trigonometric function values for angles in special right triangles to find each trigonometric value.

5. $\cos 330^\circ$

6. $\cot 225^\circ$

7. $\sin (-120^\circ)$

8. $\sec 135^\circ$

9. $\csc \frac{7\pi}{6}$

10. $\tan \left(-\frac{5\pi}{6} \right)$

11. $\cot \left(-\frac{3\pi}{2} \right)$

Directions: Use the circle to find the value of each trigonometric function.

13. $\cos 300^\circ$

14. $\csc 210^\circ$

15. $\cot 270^\circ$

16. $\sin (-135^\circ)$

17. $\tan 150^\circ$

18. $\sec (-60^\circ)$

19. $\cot \frac{\pi}{4}$

20. $\sin \left(-\frac{11\pi}{6} \right)$

21. $\sec \frac{\pi}{6}$

22. $\tan \left(-\frac{2\pi}{3} \right)$

23. $\cos \left(-\frac{3\pi}{4} \right)$

24. $\csc \frac{5\pi}{3}$

Name: _____

Pre-Calculus

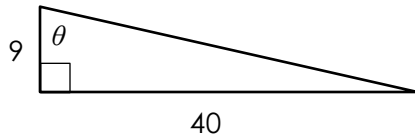
Date: _____ Per: _____

Unit 5: Trigonometric Functions

Quiz 5-2: Trigonometric Functions

Give the exact values for the trigonometric function of angle θ .

1.

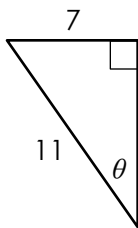


$$\sin \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}} \quad \sec \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}} \quad \csc \theta = \underline{\hspace{2cm}}$$

2.

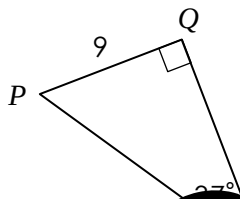


$$\sin \theta = \underline{\hspace{2cm}} \quad \cos \theta = \underline{\hspace{2cm}}$$

$$\sec \theta = \underline{\hspace{2cm}} \quad \csc \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}} \quad \cot \theta = \underline{\hspace{2cm}}$$

3. Solve the triangle below. Round measures to the nearest tenth when necessary.



$$QR = \underline{\hspace{2cm}}$$

$$PR = \underline{\hspace{2cm}}$$

$$m\angle P = \underline{\hspace{2cm}}$$

4. The angle of elevation from a football on a football field to the top of a goal post is $16^\circ 42'$. How far is the football from the base of the goal post? Round to the nearest tenth of a foot.

4. _____

5. Kayla is zip-lining in the rainforest. She is standing at the top of Platform A ready to zip-line to Platform B. If the horizontal distance between the platforms is 500 feet and the length of the zip-line is 685 feet, find the angle of depression from Platform A to Platform B to the nearest tenth.

5. _____

6. $K(-2, 4)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ .

$$\sin \theta = \underline{\hspace{2cm}} \quad \csc \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}} \quad \sec \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}} \quad \cot \theta = \underline{\hspace{2cm}}$$

7. If $\sin \theta < 0$ and $\sec \theta < 0$ in which quadrant could the terminal side of θ lie?

8. If $\csc \theta = \frac{\sqrt{6}}{2}$, and $\cot \theta < 0$, find the exact value of $\sec \theta$. 8.

9. $\tan \theta = -\frac{3}{8}$, and $\cos \theta > 0$, find the exact value of $\sin \theta$. 9.

Give the reference angle for θ .

10. 285°

12. $-\frac{11\pi}{6}$

10.

11.

12.

Give the exact value for each trigonometric function.

14. $\cot 180^\circ$

15. $\cos (-60^\circ)$

13.

14.

15.

16. $\sin \frac{\pi}{3}$

17. $\sec \left(-\frac{7\pi}{4} \right)$

18. $\tan \frac{5\pi}{6}$

16.

17.

18.

Name:

Date:

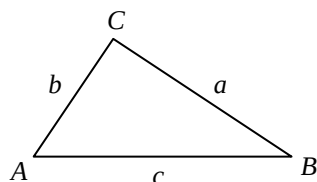
Topic:

Class:

Main Ideas/Questions

Notes/Examples

LAW OF SINES



Previously, you learned how to solve a right triangle. We now look at how to solve **oblique** (non-right) triangles.

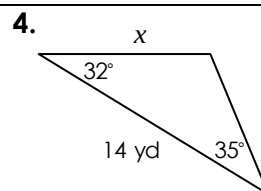
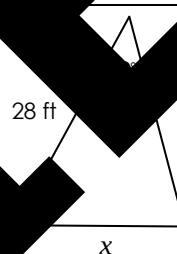
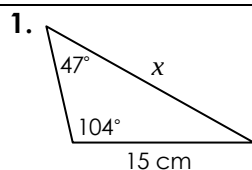
The **Law of Sines** can be used to find side and angle measures given two angles and any side (**AAS** or **ASA**), two side lengths and an angle outside of them (**SSA**).



FINDING SIDES

(Given AAS or ASA)

Directions: Find each missing measure. Round answers to the nearest tenth.



80 mi

15°

Starting Point

5. A pilot begins a flight traveling 80 miles northwest. To avoid a storm, he makes a turn, then returns to his original location. A diagram of his flight course is shown to the left. Find the total distance of the flight.

FINDING ANGLES

(Given SSA)

Watch out! When finding angles, it's possible to have one solution, two solutions, or no solution! This is called the **ambiguous case**.

Important Facts:

- The sine function always has a range between _____ and _____.
- Angles with a positive sine function value can lie in quad _____ or _____.
- To find the other possible angle, subtract from _____.
- The sum of the angle measures in a triangle is always _____.
- No triangle can have two _____ angles.

Directions: Information about $\triangle ABC$ is given below. Check your answers and give all angles.

6. $m\angle A = 60^\circ$, $AB = 30$ in, $BC = 27$ in, find $m\angle C$

7. $m\angle C = 89^\circ$, $AC = 20$ in, $AB = 28$ in, find $m\angle A$

8. $m\angle C = 102^\circ$, $AC = 21$ yd, $AB = 24$ yd, find $m\angle B$

9. $m\angle B = 95^\circ$, $AC = 34$ km, $BC = 21$ km, find $m\angle A$

10. $m\angle A = 32^\circ$, $AC = 15$ cm, $AB = 28$ cm, find $m\angle B$

11. $m\angle B = 125^\circ$, $BC = 24$ ft, $AC = 21$ ft, find $m\angle C$

Name: _____

Unit 5: Trigonometric Functions

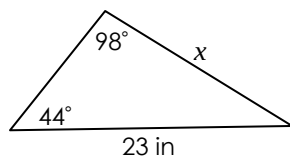
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Homework 7: Law of Sines

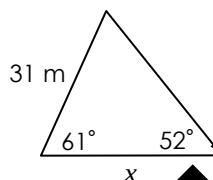
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Directions: Find each missing measure. Round answers to the nearest tenth.

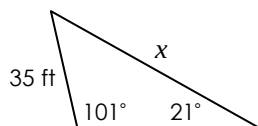
1.



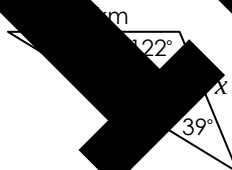
2.



3.



4.



Directions: Information about $\triangle ABC$ is given below. Check your answers and give all possibilities.

5. $m\angle A = 40^\circ$, $AB = 35$, $BC = 31$, find $m\angle C$

6. $m\angle B = 110^\circ$, $AC = 14\text{ ft}$, $BC = 7\text{ ft}$, find $m\angle A$

7. $m\angle A = 45^\circ$, $AB = 27$ mi, $BC = 16$ mi, find $m\angle C$

8. $m\angle B = 85^\circ$, $AC = 32$ in, $AB = 11$ in, find $m\angle A$

9. $m\angle C = 16^\circ$, $CB = 24$ yd, $BA = 17$ yd, find $m\angle A$

10. $m\angle B = 110^\circ$, $AC = 10$ m, $BC = 9$ m, find $m\angle A$

11. A rocket is launched from Space Coast of Florida and can be seen from local schools. The angles of elevation from Vero Beach High School and Pine High School are $41^\circ 15'$ and $23^\circ 7' 30''$ respectively. The two schools are 12 miles apart, and the rocket is launched between them, how far is the rocket from Pine High School?

Lines of crops in a garden form a triangular shape. Strawberries are planted in a 10 ft line, green beans are planted in an 18 ft line. If the third side, a line of pumpkins, makes an 68° angle with the strawberries, determine the length of the line of pumpkins.

Name:

Date:

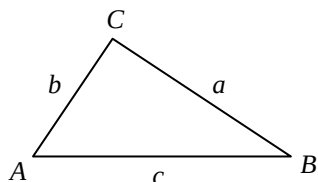
Topic:

Class:

Main Ideas/Questions

Notes/Examples

LAW OF COSINES



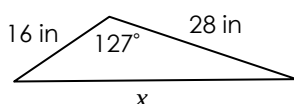
The **Law of Cosines** can be used to find side and angle measures in a triangle given two sides and the included angle (**SAS**) or three sides (**SSS**).

- _____
- _____
- _____

FINDING SIDES

Directions: Find each side length. Round answers to the nearest tenth.

1.



2.

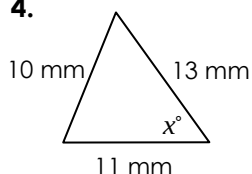


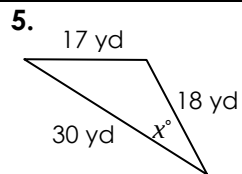
3. In $\triangle ABC$, $\angle C = 88^\circ$, $AC = 22$ ft, and $BC = 11.5$ ft. Find AB .

FINDING ANGLES

Directions: Find each angle measure. Round answers to the nearest tenth.

4.





6. In $\triangle RST$, $RS = 9$ cm, $ST = 14$ cm, and $RT = 6$ cm. Find

APPLICATIONS

Note: A *bearing* measures the acute angle that a path or line of sight makes with a fixed north-south line.

7. Two ships leave a port at 8 AM. Ship A leaves with a bearing of $N 68^\circ E$ at a speed of 17 mph. Ship B leaves with a bearing of $S 2^\circ W$ at a speed of 12 mph. If the ships maintain their paths, how far apart are they at 2 PM?

8. Bruno and Aliona are figure skaters. Starting from the same location, Bruno skates 29 feet at a bearing of $S 55^\circ 9' W$. Aliona skates 18 feet at a bearing of $S 16^\circ 1' E$. How far is Bruno now from Aliona?

9. A helicopter spots two landing pads below. The straight-line distance from the helicopter to the pads is 14 miles to Landing Pad A and 8 miles to Landing Pad B. If the landing pads are 20 miles apart, find the angle of depression from the helicopter to Landing Pad B.

Name: _____

Unit 5: Trigonometric Functions

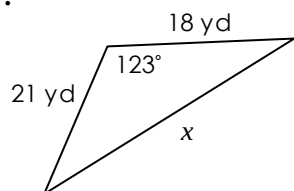
Date: _____ Per: _____

Homework 8: Law of Cosines

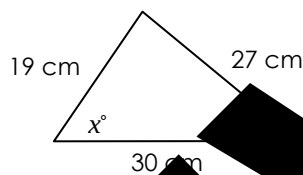
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Directions: Find the side length or angle measure. Round answers to the nearest tenth.

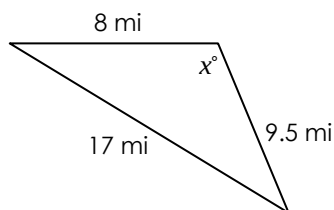
1.



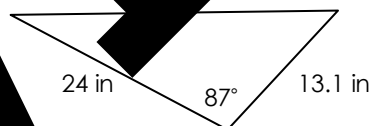
2.



3.



4.



5. In $\triangle XYZ$, $m\angle X = 115^\circ$, $XZ = 20$ in, and $XY = 20$ in.

6. In $\triangle KLM$, $KL = 15$ m, $KM = 22$ m, and $LM = 9$ m. Find $m\angle M$.

7. In $\triangle FGH$, $GH = 26$ mi, $FH = 30.8$ mi, and $FG = 13$ mi. Find $m\angle G$.

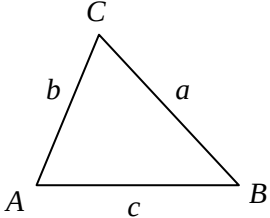
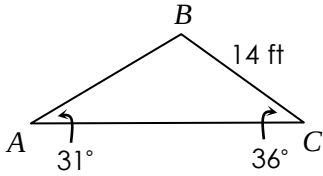
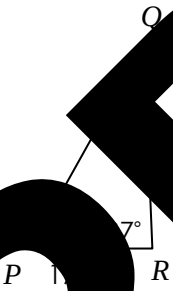
8. In $\triangle PQR$, $m\angle Q = 62^\circ$, $PQ = 15$ yd, and $RQ = 19$ yd. Find PR .

9. Michael leaves home and drives 17 miles west to work. After work, he travels 9 miles to a friend's house. If the bearing from work to the friend's house is $S 23^\circ E$, find the distance between the friend's house and home.

10. Two trucks, 36 feet apart, are towing a large car out of a snowdrift. If the lengths of the tow ropes are 27 feet and 19 feet, find the angle formed by the two tow ropes.

The cities of Dallas, Charleston, and Indianapolis form a triangle. The distance from Dallas to Charleston is 980 miles, Charleston to Indianapolis is 595 miles, and Indianapolis to Dallas is 764 miles. If Dallas and Charleston lie on the same latitude, find the bearing from Charleston to Indianapolis.

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
LAW OF SINES & LAW OF COSINES 	LAW OF SINES	LAW OF COSINES
	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Use when given:	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$ Use when:
EXAMPLE 1: 	Directions: Use the law of sines and/or law of cosines to solve each triangle.	
EXAMPLE 2: 		
EXAMPLE 3: In $\triangle DEF$, $\angle D = 42^\circ$, $\angle E = 73^\circ$, $DE = 6$ in, $DF = 14$ in, and $DF = 13$ in.		
		$AB =$
		$AC =$
		$m\angle B =$
		$PQ =$
		$m\angle P =$
		$m\angle Q =$
		$m\angle D =$
		$m\angle E =$
		$m\angle F =$

EXAMPLE 4:In $\triangle WXY$: $WX = 9$ m, $XY = 15$ m,and $m\angle W = 31^\circ$ $WY =$ $m\angle X =$ $m\angle Y =$ **APPLICATIONS**

5. Cincinnati, Ohio is 540 miles due east of Kansas City, Missouri. The bearing from Kansas City to Nashville, Tennessee is $S 61^\circ E$. The bearing from Cincinnati to Nashville is $S 29^\circ W$.

a) Find the distance from Kansas City to Nashville.

b) Find the distance from Cincinnati to Nashville.

Two ships leave from the same port, Ship A sails 8 miles at a bearing of $N 45^\circ W$ and Ship B sails 4 miles at a bearing of $S 70^\circ W$.

a) If Ship B bears north of a mile, how far is Ship A from Ship B?

b) If Ship A sails to Ship B, what bearing should it take?

Name:

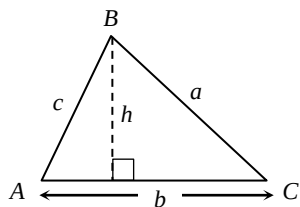
Date:

Topic:

Class:

Main Ideas/Questions

AREA OF TRIANGLES



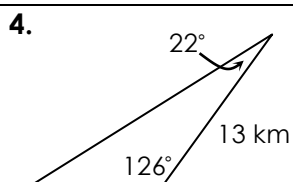
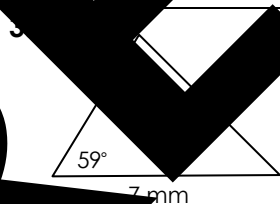
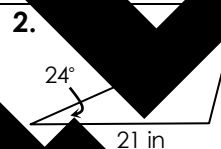
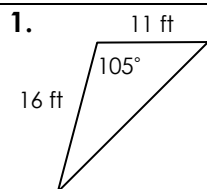
Notes/Examples

- What is the formula for the area of a triangle? _____
- We know that $\sin C =$ _____
- Therefore, $h =$ _____

Substitute this into the area formula for _____

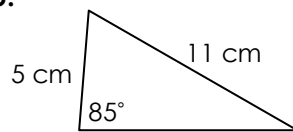
EXAMPLES

Directions: Find the area of each triangle and round to the nearest tenth.

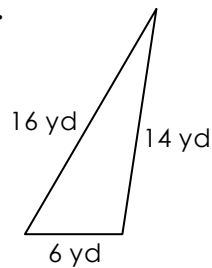


Note: You need **SAS** to find the area. Use the Law of Sines or the Law of Cosines first for these examples.

5.



6.

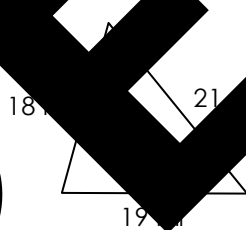


HERON'S FORMULA

When the measures of the three sides of a triangle are given, you can use **Heron's Formula** to find the area:

where $s = \frac{a + b + c}{2}$

EXAMPLES



8. A triangular-shaped piece of glass has side lengths that measure 35 inches, 27 inches, and 14 inches. Find the area of the glass to the nearest tenth of a square inch.

Name: _____

Unit 5: Trigonometric Functions

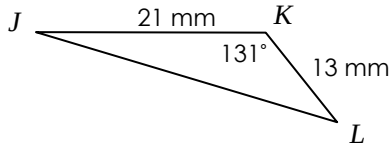
Date: _____ Per: _____

Homework 9: Law of Sines & Law of Cosines;
Area of Triangles

**** This is a 2-page document! ****

Directions: Use the Law of Sines and/or the Law of Cosines to solve each triangle. Round to the nearest tenth when necessary.

1.

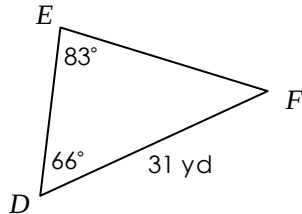


$$JL =$$

$$m\angle J =$$

$$m\angle L =$$

2.

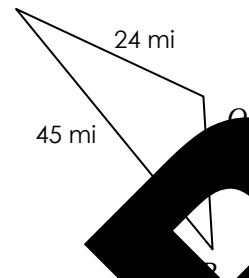


$$DE =$$

$$EF =$$

$$m\angle F =$$

3.



$$m\angle P =$$

$$m\angle Q =$$

$$m\angle R =$$

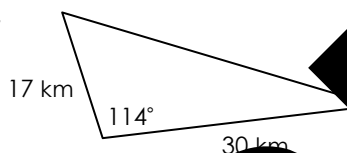
4. Jason and Allison are hiking in the woods when they spot a rare owl in a tree. Jason stops and measures an angle of elevation of $22^\circ 8' 6''$. At the same time, Allison is standing 48 feet closer to the tree and measures an angle of elevation of $30^\circ 40' 30''$ to the owl. If Jason and Allison are the same height, their eyes 5 feet from the ground, find the height of the owl in the tree.

5. A drone is flying over a college campus. The drone spots the cafeteria on the west end of campus at a 27° angle of depression and the sports complex on the east end of campus at a 34° angle of depression. If the cafeteria and sports complex are separated by a straight road 2.3 mi long, how high is the drone?

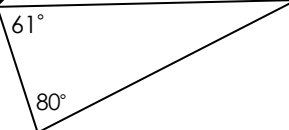
6. Two bicyclists left at 11:30 AM for a ride. Rider A traveled at a constant rate of 12 mph at a bearing of N 47° E, stopping at 1:30 PM. Rider B traveled until 2:00 PM, riding at a constant rate of 9 mph at a bearing of S 62° E. Determine the distance between the two bicyclists.

Directions: Find the area of each triangle. Round to the nearest tenth.

7.



17 ft



In $\triangle ABC$, $AB = 26$ m, $\angle A = 127^\circ$, and $\angle B = 49^\circ$. Find its area.

10. If a triangular garden has sides of 8 ft, 9 ft 3 in, and 13 ft 9 in, use Heron's Formula to find its area.

Name: _____

Pre-Calculus

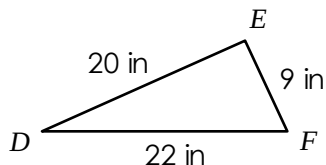
Date: _____ Per: _____

Unit 5: Trigonometric Functions

Quiz 5-3: Law of Sines & Law of Cosines

Solve each triangle using the Law of Sines and Law of Cosines. Round measures to the nearest tenth.

1.

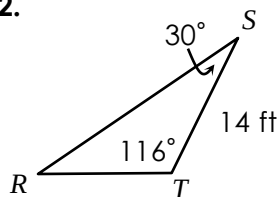


$m\angle D =$ _____

$m\angle E =$ _____

$m\angle F =$ _____

2.

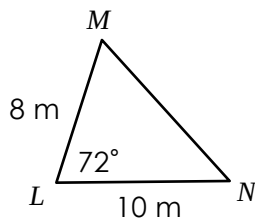


$m\angle R =$ _____

$RT =$ _____

$m\angle R =$ _____

3.



$MN =$ _____

$m\angle M =$ _____

$m\angle N =$ _____

Using the description of $\triangle ABC$, give all possible solutions for the angle measure.

4. $m\angle C = 105^\circ$, $AC = 10$ ft, $BC = 12$ ft, find $m\angle B$

4. $m\angle B =$ _____

5. $m\angle C = 95^\circ$, $BC = 30$ yd, $AC = 26$ yd, find $m\angle A$

5. $m\angle A =$ _____

6. A blimp is flying directly above a football field. The angle of depression to the base of one goal post is 40° . The angle of depression to the base of the other goal post is 72° . If the goal posts are 360 feet apart, how high is the blimp flying? Round to the nearest tenth of a foot.

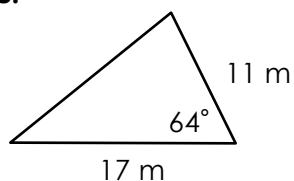
6. _____

7. _____

7. The bearing from a lighthouse to Ship A is $S 71^\circ E$. The bearing from the lighthouse to Ship B is $S 39^\circ E$. If Ship A is 15 kilometers from the lighthouse and Ship B is 10 kilometers from the lighthouse, find the distance between the ships.

Find the area of each triangle. Round answers to the nearest $\frac{1}{2}$ in.

8.



8. _____

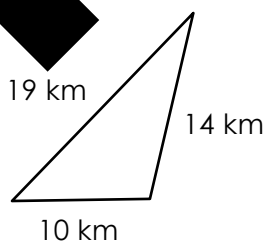
9. _____

10. _____

9.



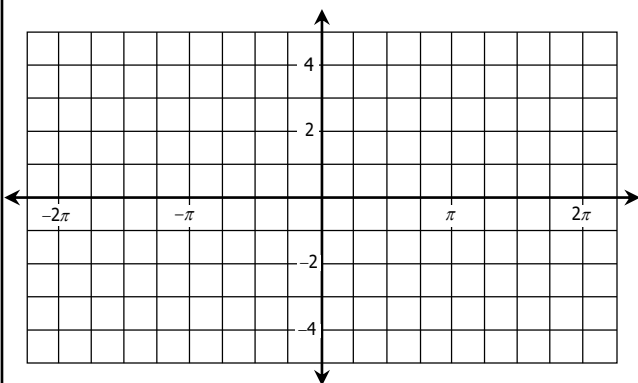
10.



Trigonometric Functions

SINE

Parent Function:



Domain:

Range:

x-intercept(s):

y-intercept:

Asymptotes:

Amplitude:

Period:

Transformations:

Amplitude:

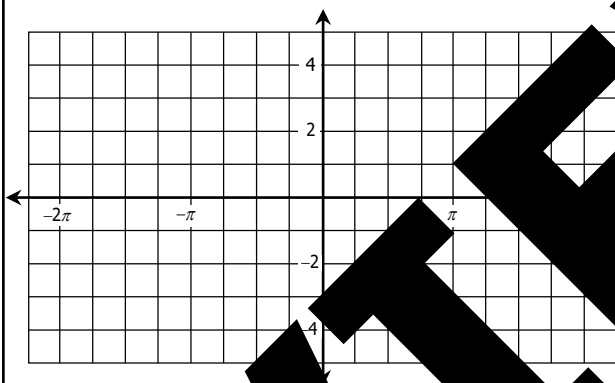
Period:

Horizontal (Phase) Shift:

Vertical Shift:

COSINE

Parent Function:



Domain:

Range:

x-intercept(s):

y-intercept:

Asymptotes:

Amplitude:

Period:

Transformations:

Amplitude:

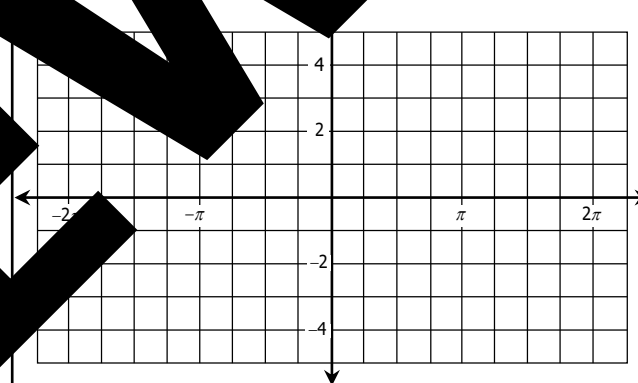
Period:

Horizontal (Phase) Shift:

Vertical Shift:

TANGENT

Parent Function:



Domain:

Range:

x-intercept(s):

y-intercept:

Asymptotes:

Amplitude:

Period:

Transformations:

Amplitude:

Period:

Horizontal (Phase) Shift:

Vertical Shift:

Name:

Date:

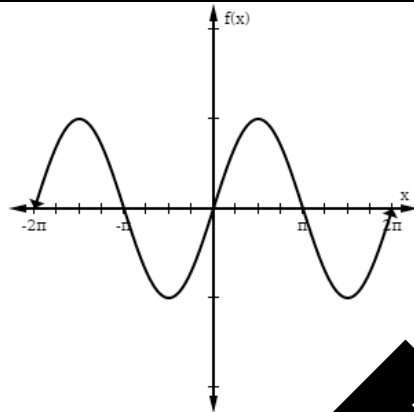
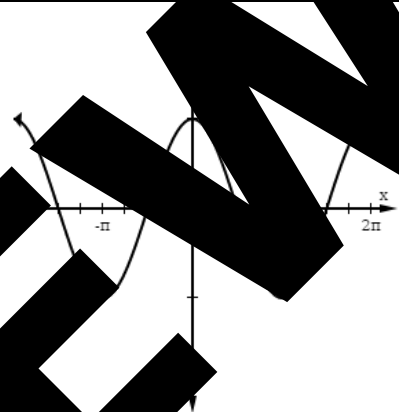
Topic:

Class:

Main Ideas/Questions

Notes/Examples

Graphing
SINE & COSINE
Functions

SINE $f(x) = \sin x$ **COSINE** $f(x) = \cos x$ 

Notice that the cosine function is a horizontal reflection of the sine function. Any transformation of a sine function is called a **sinusoid**. Therefore, sine and cosine functions are called **sinusoidal functions**.

Functions that repeat at regular intervals are called **periodic functions**.

Period:**Amplitude:**

Given $f(x) = a \cdot \sin bx$ or $f(x) = a \cdot \cos bx$,

the amplitude is _____ and the period is _____.

Identify the amplitude and period of each function.

1. $f(x) = \sin 4x$

2. $f(x) = \frac{7}{2} \cdot \cos x$

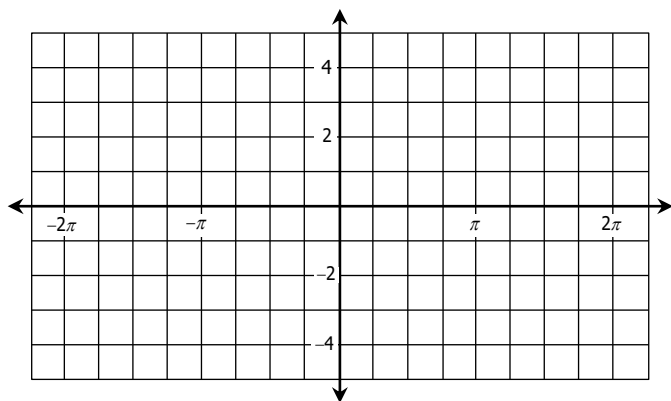
3. $f(x) = -5 \cdot \cos \frac{1}{3}x$

4. $f(x) = -2 \cdot \sin \frac{4}{3}x$

Directions: Identify the amplitude and period, then graph the function.

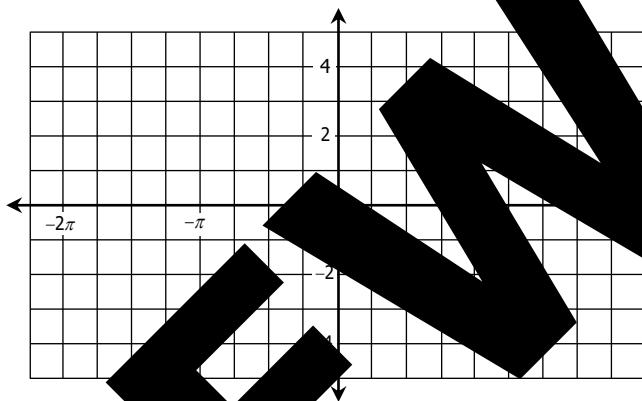
5. $f(x) = 2 \cdot \sin x$

Amplitude: _____ Period: _____



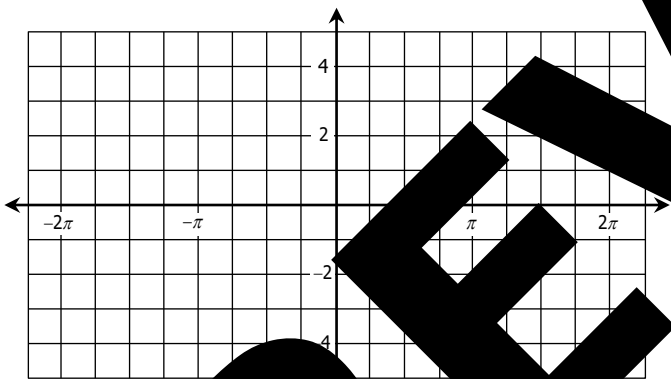
6. $f(x) = 5 \cdot \cos \frac{1}{2}x$

Amplitude: _____ Period: _____



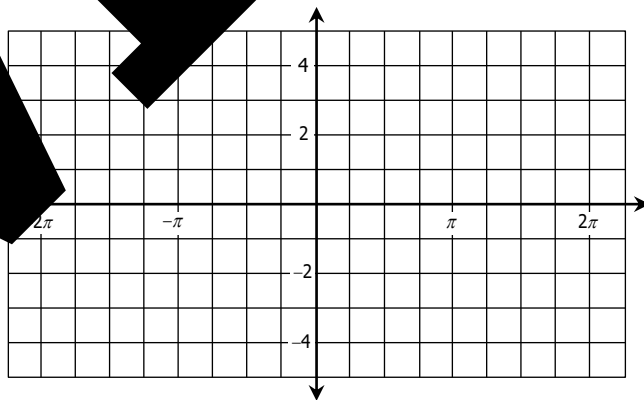
7. $f(x) = -\frac{3}{2} \cdot \cos 4x$

Amplitude: _____ Period: _____



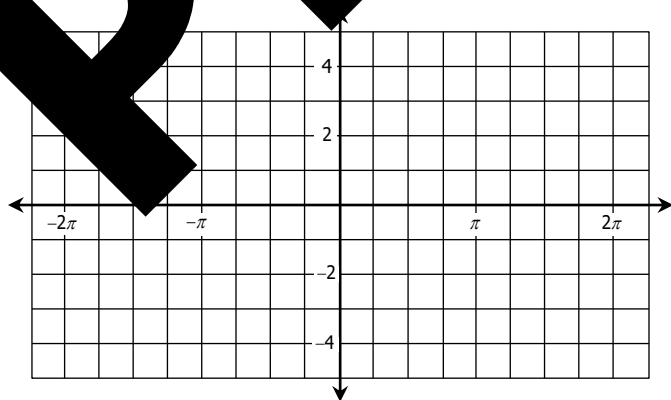
8. $f(x) = -\sin \frac{1}{3}x$

Amplitude: _____ Period: _____



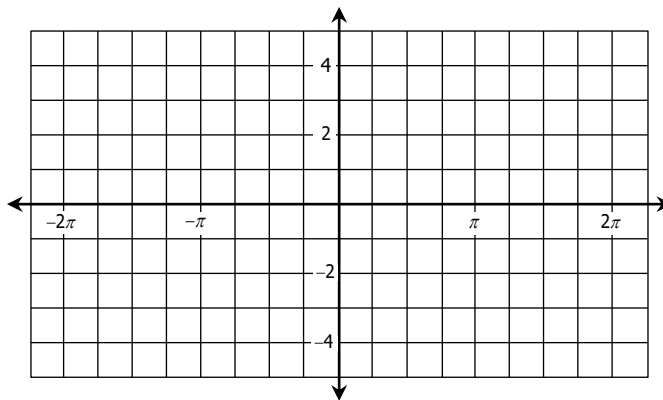
9. $f(x) = -\frac{2}{3} \cdot \sin x$

Amplitude: _____ Period: _____



10. $f(x) = 4 \cdot \sin 2x$

Amplitude: _____ Period: _____

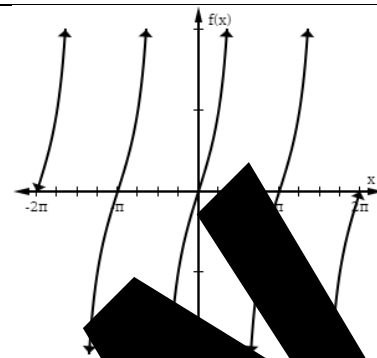


Graphing TANGENT Functions

TANGENT $f(x) = \tan x$

The tangent function is undefined for certain values of x . Therefore, the tangent function has **vertical asymptotes**.

The tangent function does not have an amplitude.

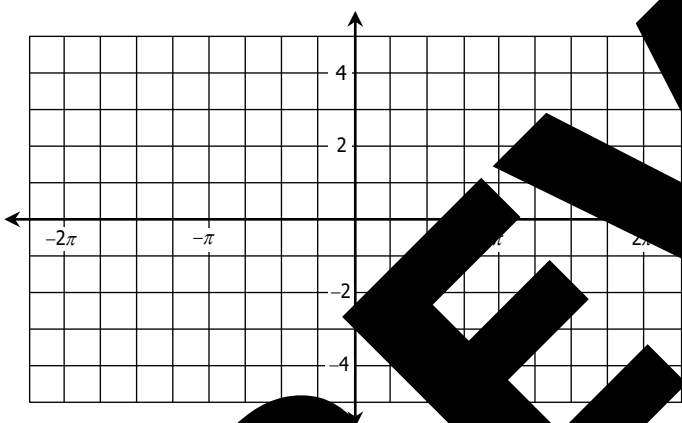


Given $f(x) = a \cdot \tan bx$, the period is _____ and the asymptotes are _____ multiples of _____.

Directions: Identify the period, then graph each function. Show the asymptotes on the graph.

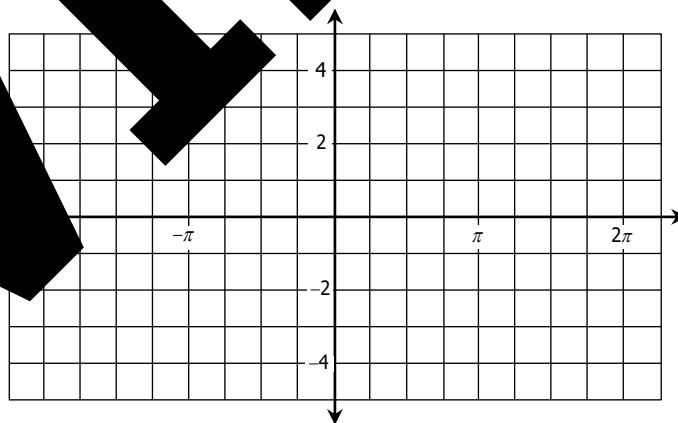
11. $f(x) = 4 \cdot \tan x$

Period: _____



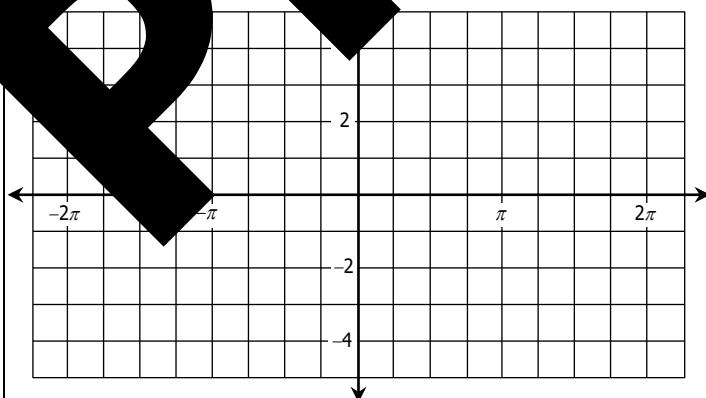
12. $f(x) = -\tan x$

Period: _____



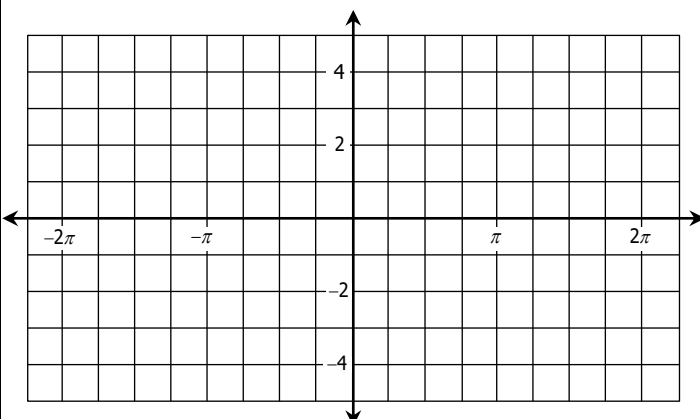
13. $f(x) = 5 \cdot \tan 2x$

Period: _____



14. $f(x) = 3 \cdot \tan \frac{2}{3}x$

Period: _____



Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

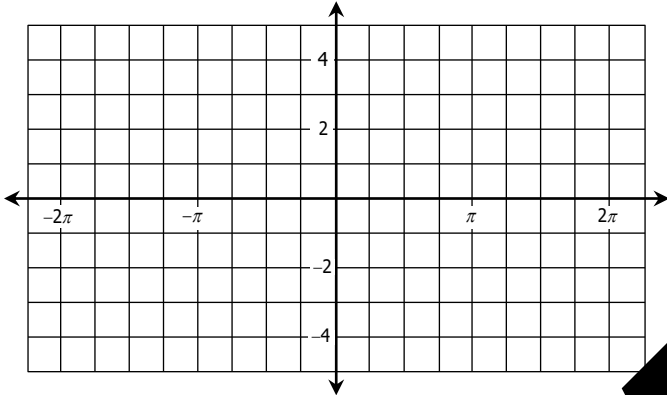
Homework 10: Graphing Sine, Cosine, & Tangent Functions

**** This is a 2-page document! ****

Directions: Identify the amplitude and period, then graph the function.

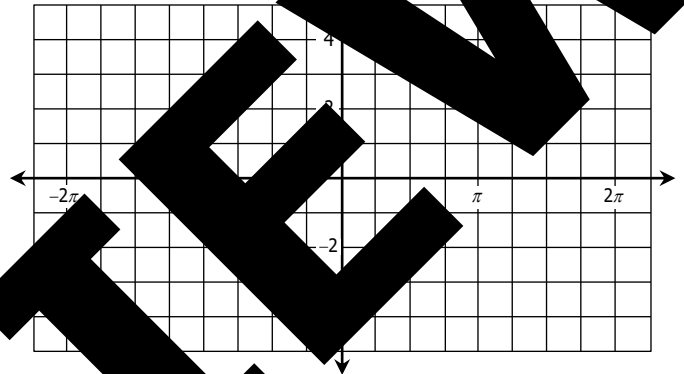
1. $f(x) = -2 \cdot \cos x$

Amplitude: _____ Period: _____



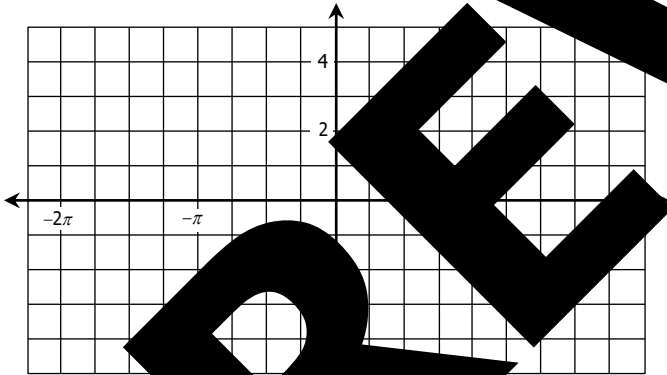
2. $f(x) = \frac{5}{2} \cdot \sin 2x$

Amplitude: _____ Period: _____



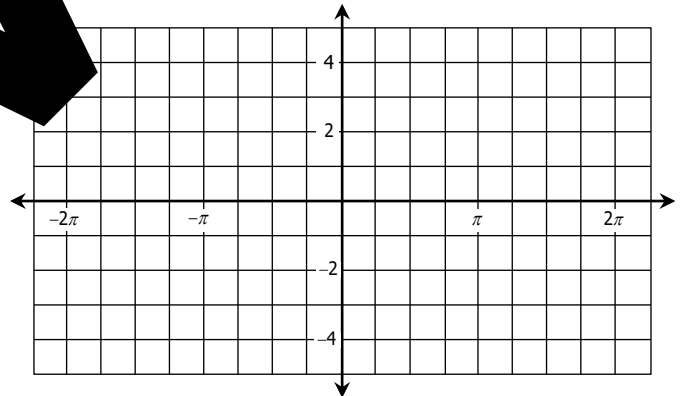
3. $f(x) = \sin \frac{2}{3}x$

Amplitude: _____ Period: _____



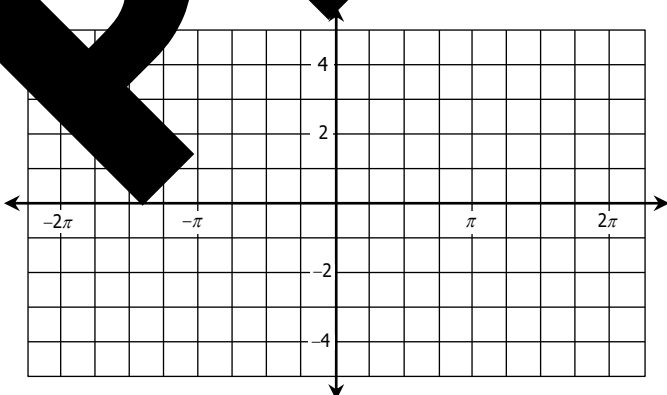
4. $f(x) = 2 \cdot \sin 8x$

Amplitude: _____ Period: _____



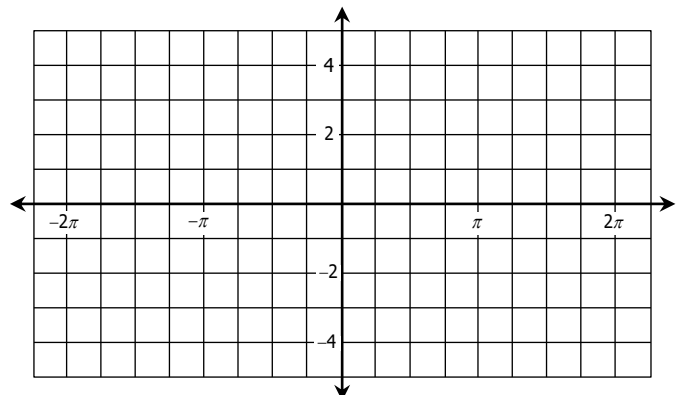
5. $f(x) = \sin \frac{4}{3}x$

Amplitude: _____ Period: _____



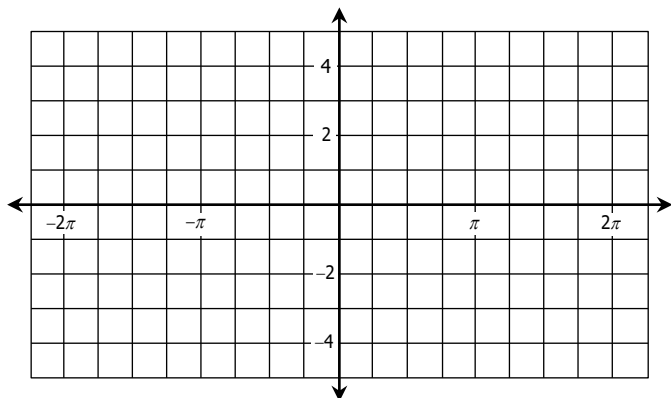
6. $f(x) = \cos 2x$

Amplitude: _____ Period: _____



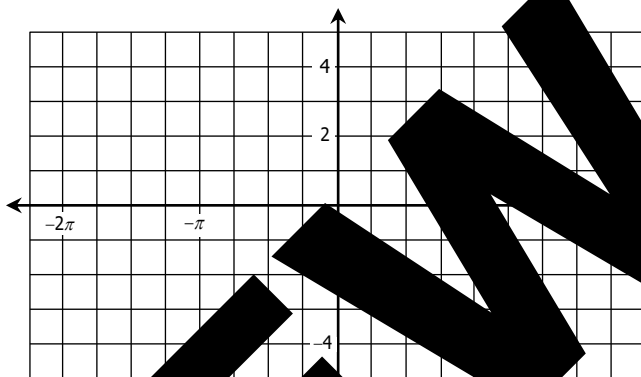
7. $f(x) = -\frac{7}{2} \cdot \cos \frac{1}{2}x$

Amplitude: _____ Period: _____



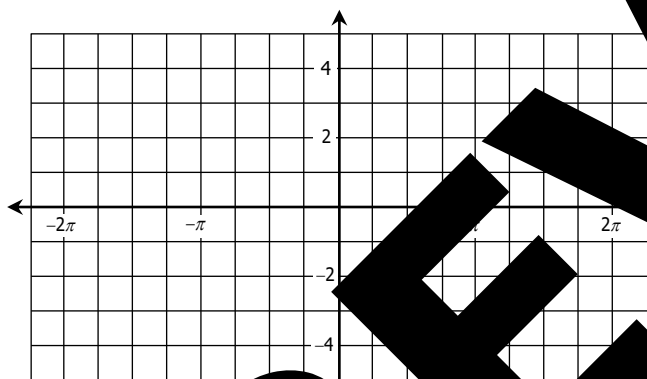
8. $f(x) = \frac{1}{2} \cdot \sin 4x$

Amplitude: _____ Period: _____



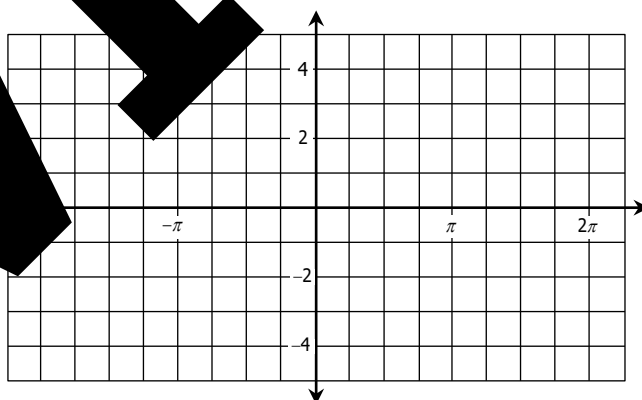
9. $f(x) = 2 \tan x$

Amplitude: _____ Period: _____



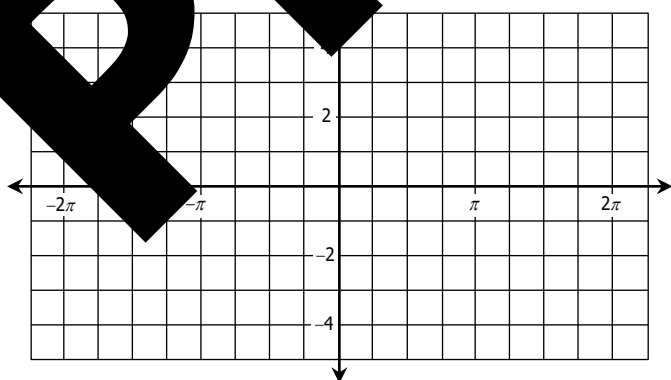
10. $f(x) = 3 \tan \frac{1}{3}x$

Amplitude: _____ Period: _____



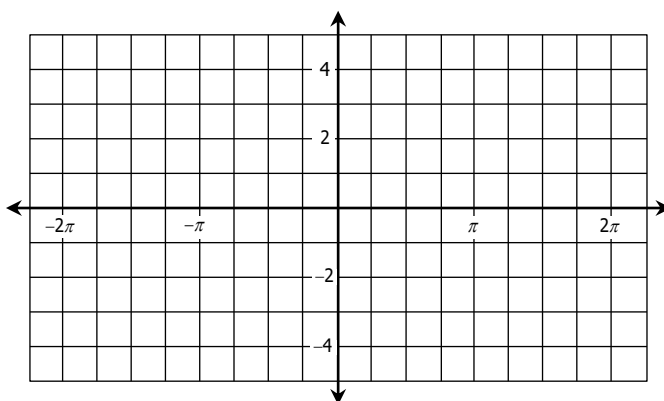
11. $f(x) = \frac{1}{2} \tan \frac{1}{2}x$

Amplitude: _____ Period: _____

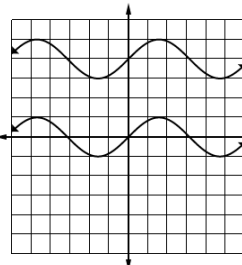
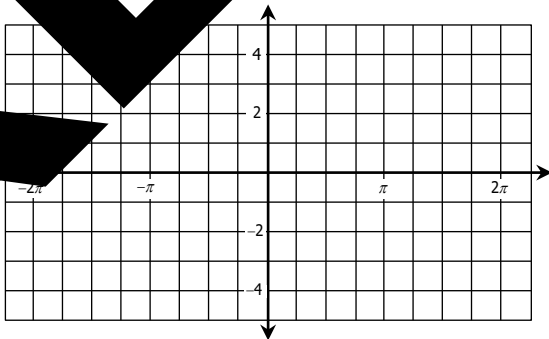
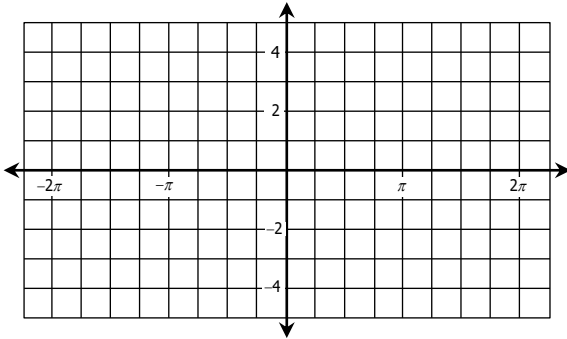


12. $f(x) = -\frac{3}{2} \tan \frac{1}{2}x$

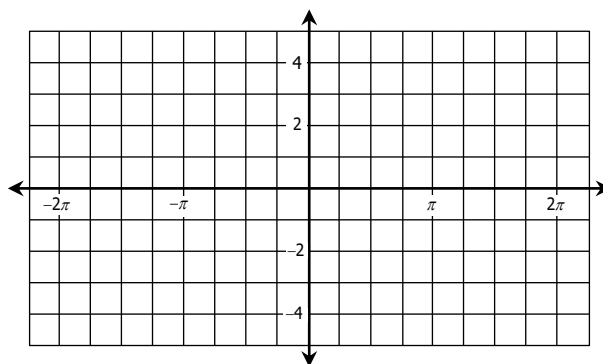
Amplitude: _____ Period: _____



Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
TRANSLATIONS of Trigonometric Functions $f(x) = a \cdot \sin b(x - h) + k$ $f(x) = a \cdot \cos b(x - h) + k$ $f(x) = a \cdot \tan b(x - h) + k$	Like other functions, trigonometric functions can be translated. Given the functions to the left, h represents a _____ and k represents a _____.	
	Vertical Shifts	Horizontal Shifts
	$+k$ shifts _____ $-k$ shifts _____	h shifts _____ $-h$ shifts _____
	Describe the the translations of each trigonometric function:	
	1. $f(x) = \cos x + 2$	2. $f(x) = \tan x - 3$
3. $f(x) = \sin\left(x + \frac{\pi}{2}\right)$	4. $f(x) = \cos(x - \pi)$	
MIDLINE	- Shifting a graph vertically will create a new horizontal axis, called _____. - This is the reference line which a graph oscillates around. The equation for the midline is _____.	
		
5. $f(x) = \frac{3}{2} \cdot \sin x + 3$	Graph the function and identify each characteristic.	
		Amplitude:
		Period:
		Phase Shift:
		Vertical Shift:
		Midline:
6. $f(x) = \tan\left(x + \frac{\pi}{2}\right)$		Amplitude:
Period:		
Phase Shift:		
Vertical Shift:		
Midline:		

7. $f(x) = 4 \cdot \cos \frac{2}{3}(x - \pi) + 1$



Amplitude:

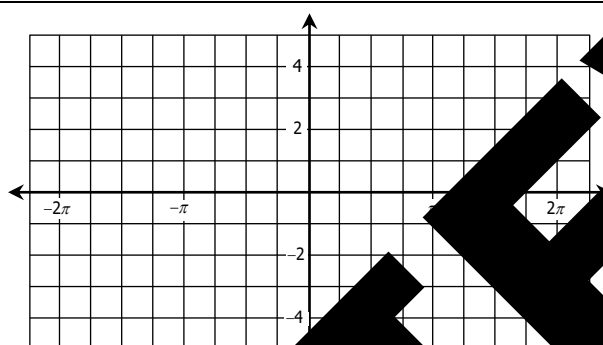
Period:

Phase Shift:

Vertical Shift:

Midline:

8. $f(x) = \tan 2\left(x + \frac{\pi}{4}\right) - 3$



Amplitude:

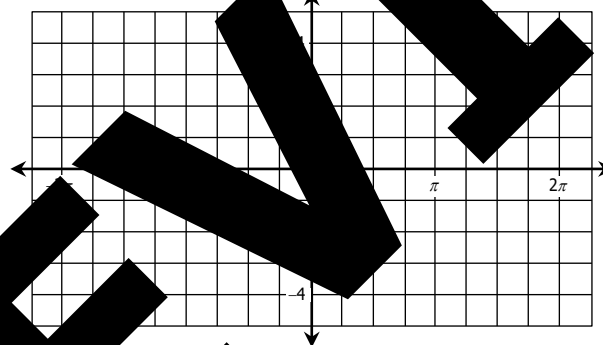
Period:

Phase Shift:

Vertical Shift:

Midline:

9. $f(x) = 3 \cdot \sin 4\left(x - \frac{\pi}{2}\right) - 2$



Amplitude:

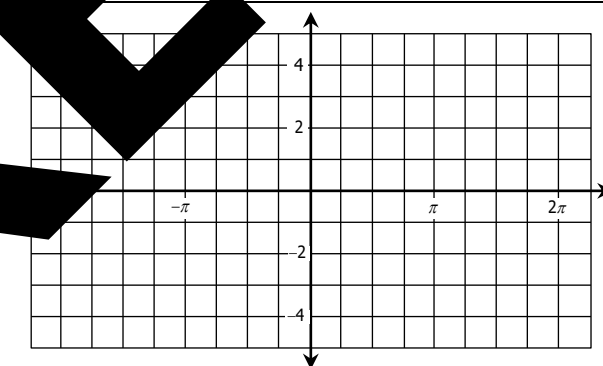
Period:

Phase Shift:

Vertical Shift:

Midline:

10. $f(x) = \cos 2\left(x + \frac{3\pi}{4}\right) + 4$



Amplitude:

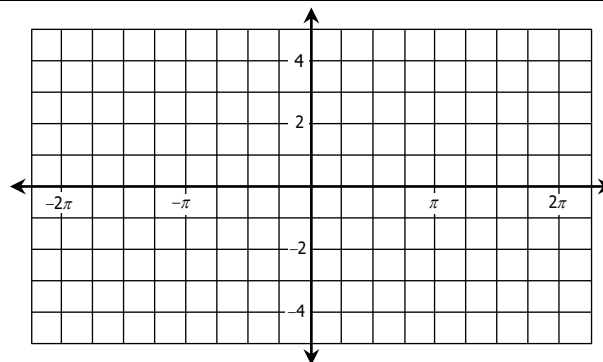
Period:

Phase Shift:

Vertical Shift:

Midline:

11. $f(x) = 2 \cdot \tan \frac{1}{2}(x + \pi) + 1$



Amplitude:

Period:

Phase Shift:

Vertical Shift:

Midline:

Name: _____

Unit 5: Trigonometric Functions



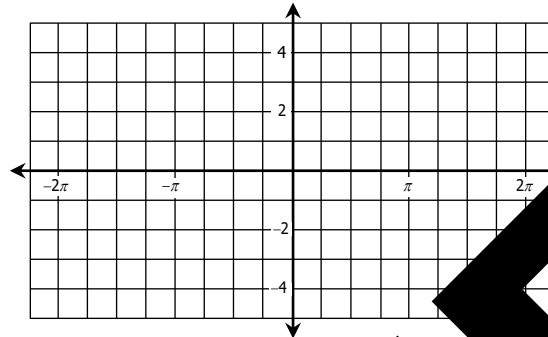
Date: _____ Per: _____

Homework 11: Translating Trigonometric Functions

** This is a 2-page document! **

Directions: Graph the function, then identify each characteristic.

1. $f(x) = 2 \cdot \cos(x + \pi)$



Amplitude:

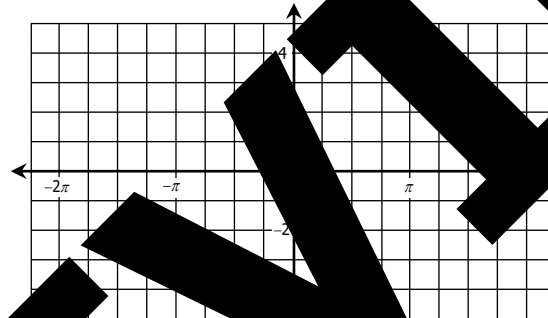
Period:

Phase Shift:

Vertical Shift:

Midline:

2. $f(x) = \sin x + 4$



Amplitude:

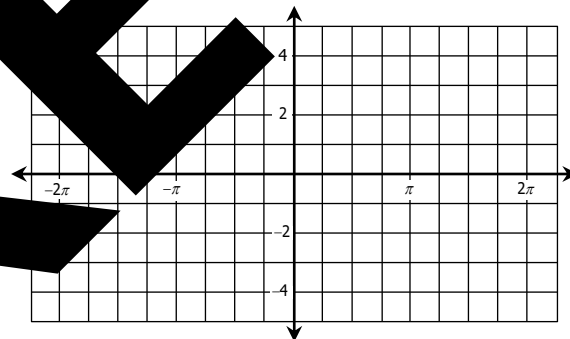
Period:

Phase Shift:

Vertical Shift:

Midline:

3. $f(x) = 2 \cdot \tan 2\left(x - \frac{3\pi}{4}\right)$



Amplitude:

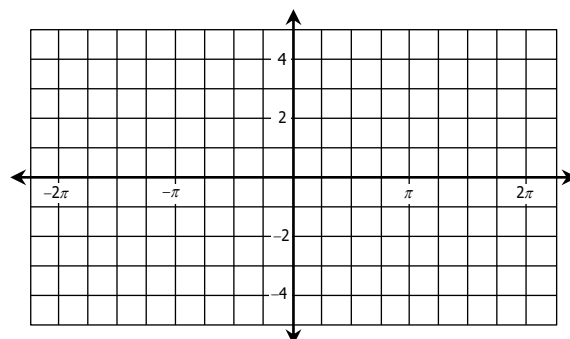
Period:

Phase Shift:

Vertical Shift:

Midline:

4. $f(x) = \frac{1}{2}\left(x - \frac{\pi}{2}\right) - 1$



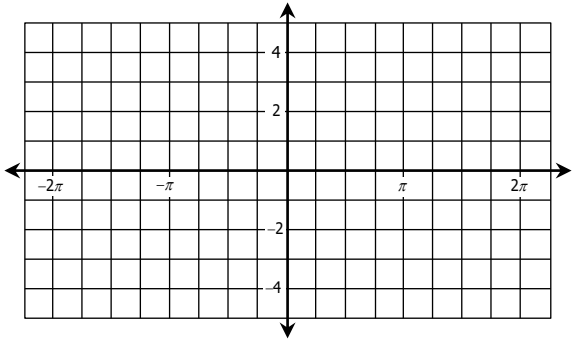
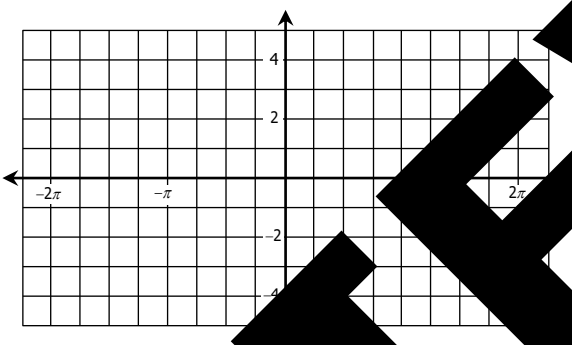
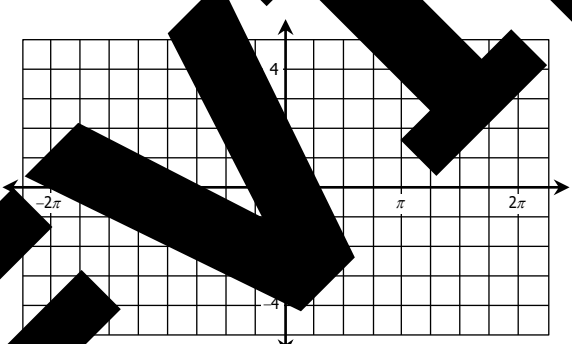
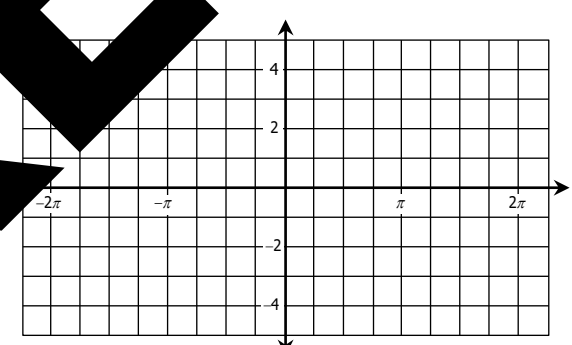
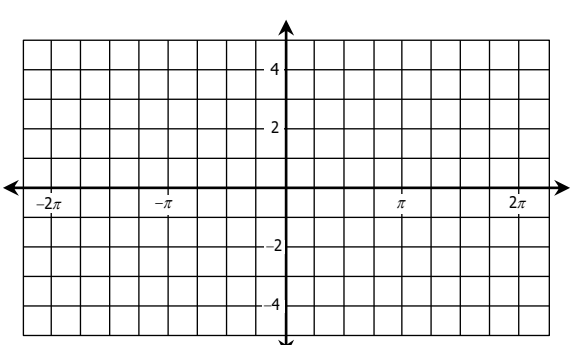
Amplitude:

Period:

Phase Shift:

Vertical Shift:

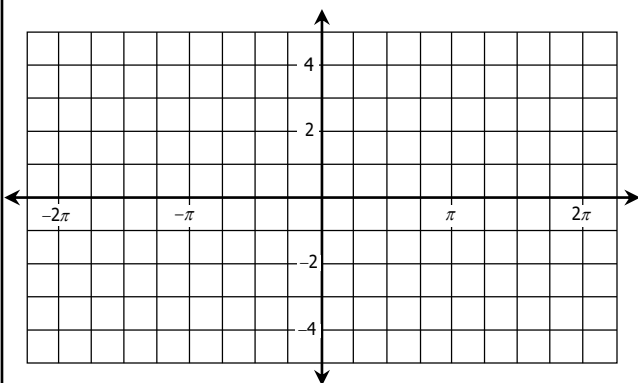
Midline:

5. $f(x) = \sin \frac{4}{3} \left(x + \frac{3\pi}{2} \right) + 2$		Amplitude:
6. $f(x) = \frac{3}{2} \cdot \cos \frac{2}{3} \left(x - \frac{\pi}{4} \right)$		Period:
7. $f(x) = 3 \cdot \tan \frac{1}{3} (x + \pi) + 1$		Phase Shift:
8. $f(x) = -2 \cdot \cos 2 \left(x + \frac{5\pi}{2} \right) - 3$		Vertical Shift:
9. $f(x) = \sin \frac{2}{3} \left(x - \frac{\pi}{2} \right) - 1$		Midline:
		Amplitude:
		Period:
		Phase Shift:
		Vertical Shift:
		Midline:

Trigonometric Functions

COSECANT

Parent Function:



Domain:

Range:

x-intercept(s):

y-intercept:

Asymptotes:

Amplitude:

Period:

Transformations:

Amplitude:

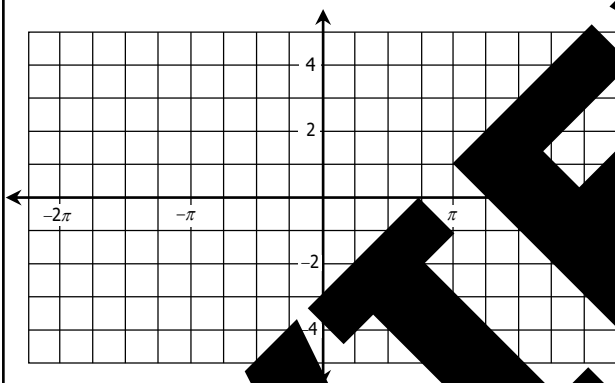
Period:

Horizontal (Phase) Shift:

Vertical Shift:

SECANT

Parent Function:



Domain:

Range:

x-intercept(s):

y-intercept:

Asymptotes:

Amplitude:

Period:

Transformations:

Amplitude:

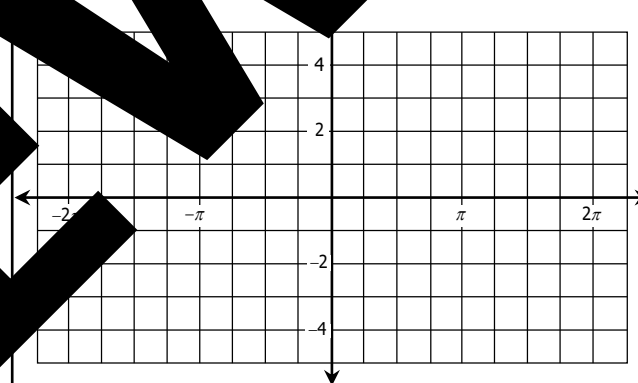
Period:

Horizontal (Phase) Shift:

Vertical Shift:

COTANGENT

Parent Function:



Domain:

Range:

x-intercept(s):

y-intercept:

Asymptotes:

Amplitude:

Period:

Transformations:

Amplitude:

Period:

Horizontal (Phase) Shift:

Vertical Shift:

Name:

Date:

Topic:

Class:

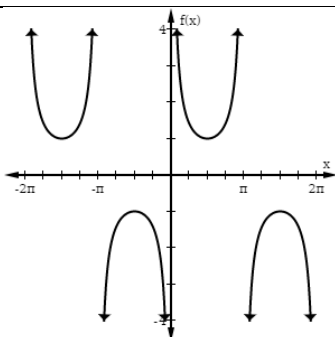
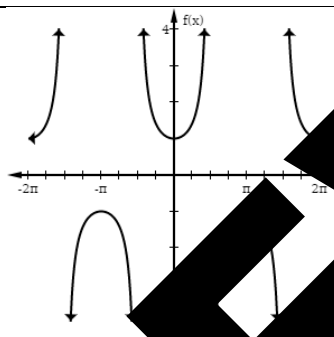
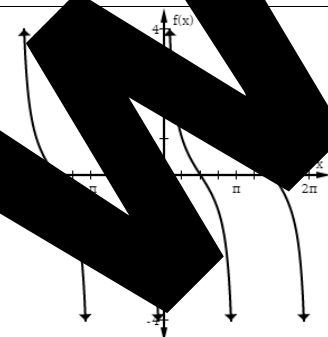
Main Ideas/Questions

Notes/Examples

Graphing RECIPROCAL Functions

*All reciprocal functions have vertical asymptotes.

*Reciprocal functions do not have an amplitude.

COSECANT $f(x) = \csc x$ SECANT $f(x) = \sec x$ COTANGENT $f(x) = \cot x$ 

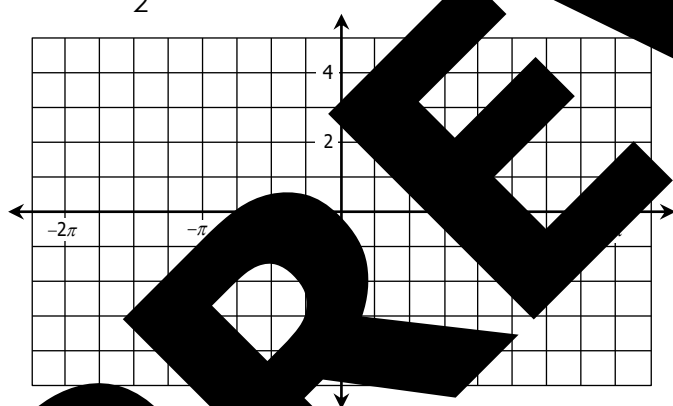
Given $f(x) = a \cdot \csc bx$, $f(x) = a \cdot \sec bx$, or $f(x) = a \cdot \cot bx$:

- The period of cosecant and secant functions is _____.
- The period of the cotangent function is _____.

Directions: Identify the period, then graph each function. Show the asymptotes on the graph.

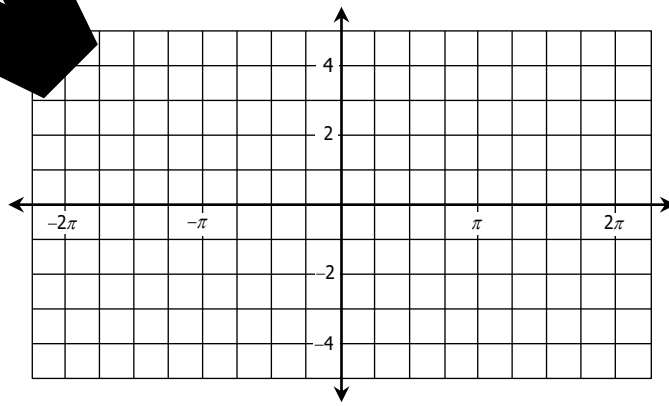
1. $f(x) = \frac{3}{2} \cdot \csc x$

Period: _____



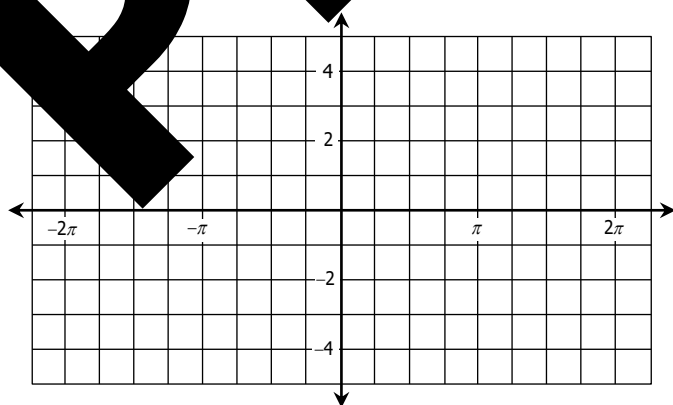
$f(x) = 2 \cdot \sec 4x$

Period: _____



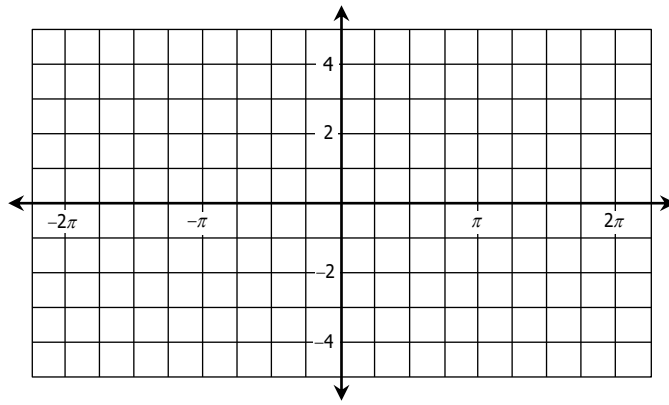
$f(x) = \frac{4}{3} \cdot \csc x$

Period: _____

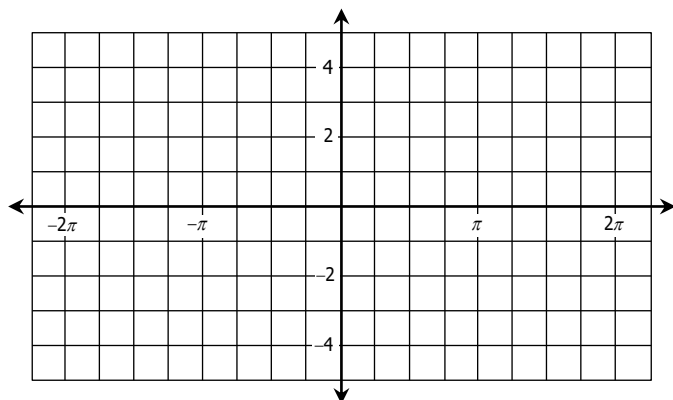


4. $f(x) = \frac{5}{2} \cdot \csc 2x$

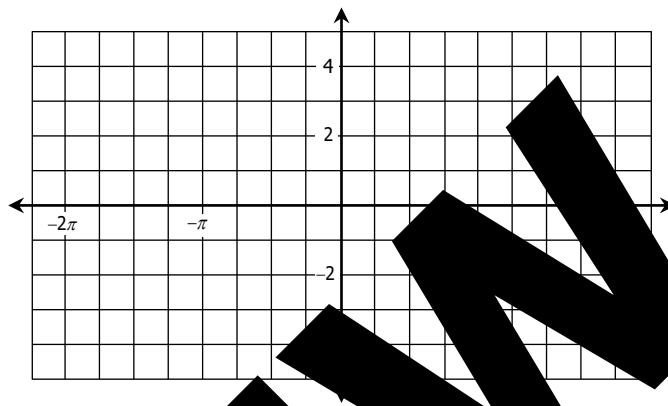
Period: _____



5. $f(x) = 3 \cdot \cot 2x$ Period: _____

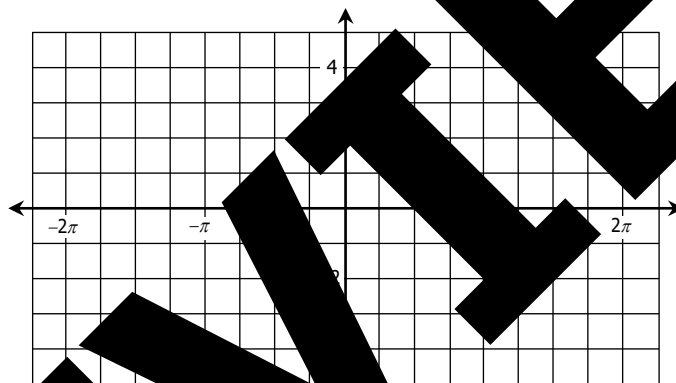


6. $f(x) = -\cot \frac{4}{3}x$ Period: _____



Directions: Graph each function and show the asymptotes on each graph. Identify the phase shift, vertical shift, and midline of each function.

7. $f(x) = \csc \left(x - \frac{\pi}{4} \right) - 2$



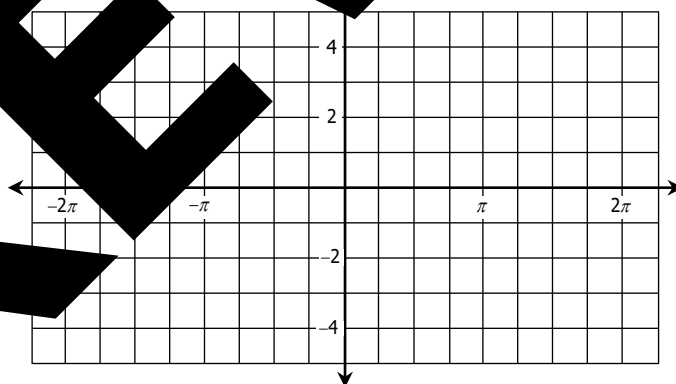
Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

8. $f(x) = 2 \cdot \sec \frac{8}{3} \left(x + \pi \right) + 1$



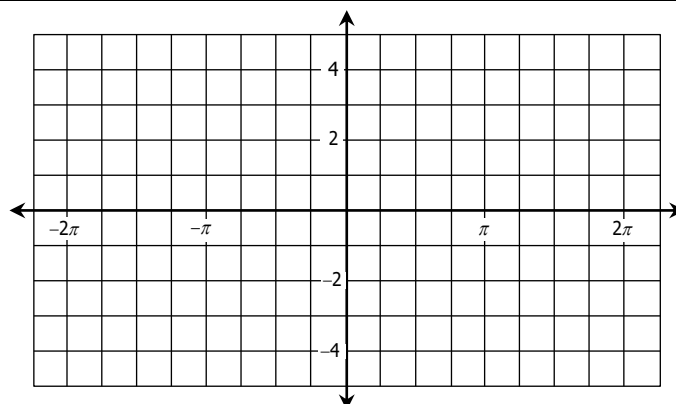
Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

9. $f(x) = -2 \left(x - \frac{3\pi}{2} \right) - 3$



Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

Name: _____

Unit 5: Trigonometric Functions

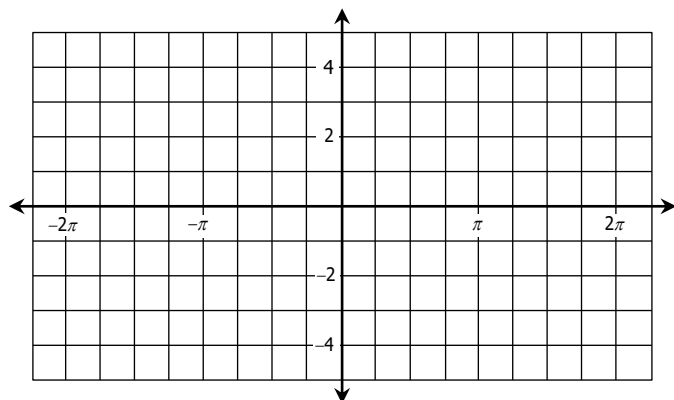
Date: _____ Per: _____

Homework 12: Graphing Reciprocal Functions

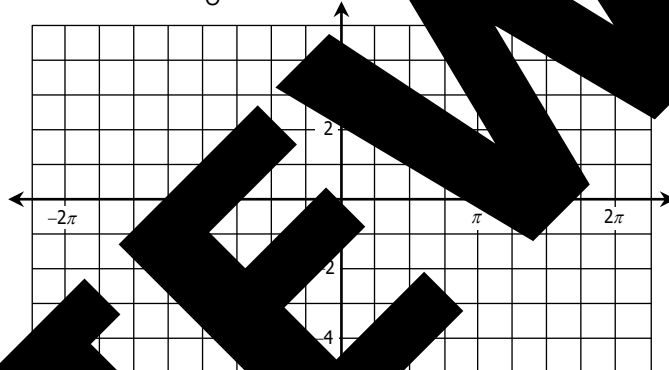
**** This is a 2-page document! ******Directions:** Identify the period, then graph each function. Show the asymptotes on the graph.

1. $f(x) = \sec 2x$

Period: _____

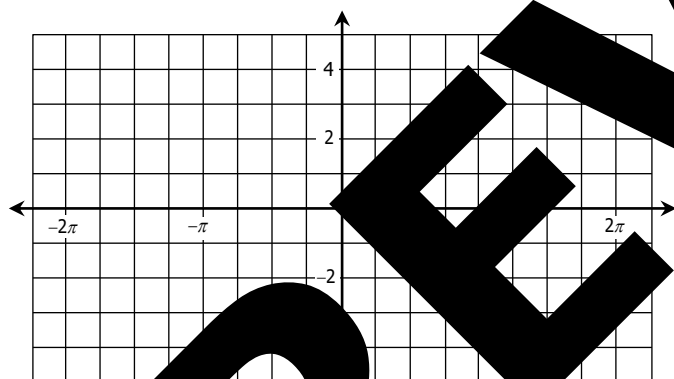


2. $f(x) = 4 \cdot \cot \frac{2}{3}x$



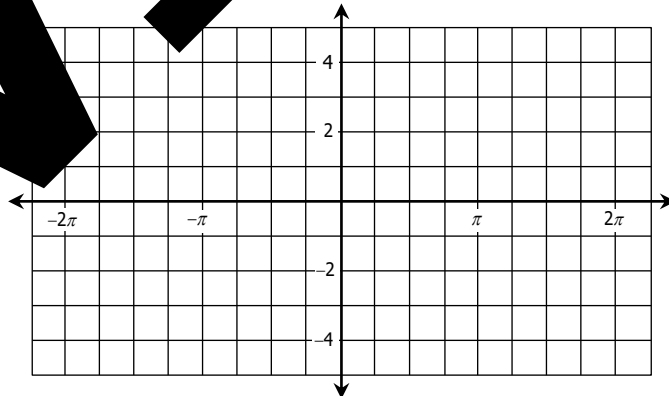
3. $f(x) = \frac{7}{2} \cdot \csc \frac{4}{3}x$

Period: _____



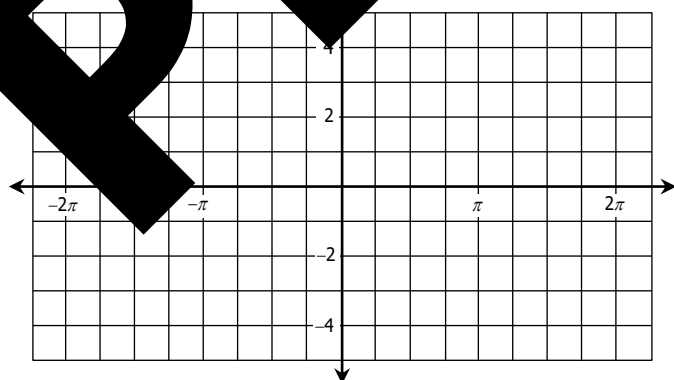
4. $f(x) = 3 \cdot \cot x$

Period: _____



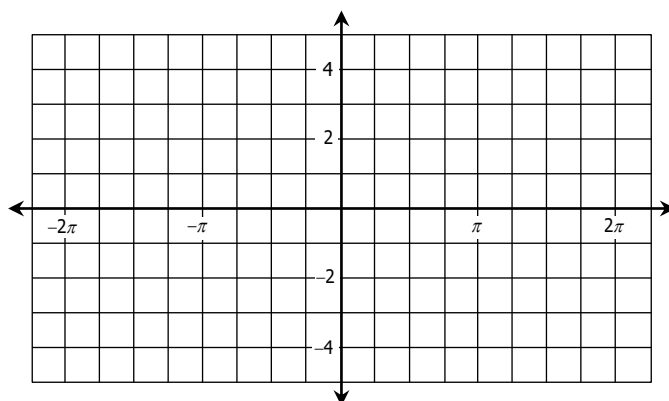
5. $f(x) = \cot \frac{1}{5}x$

Period: _____



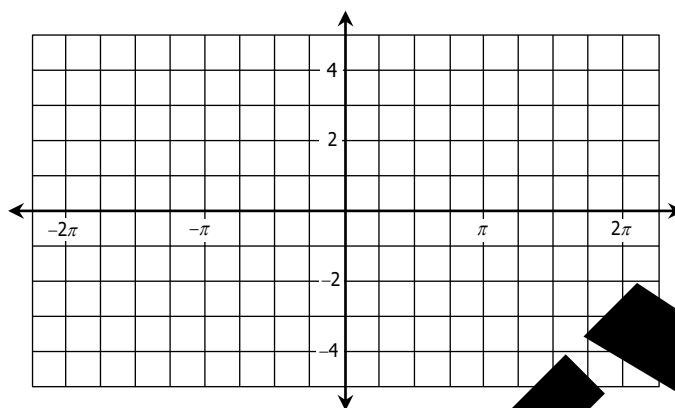
6. $f(x) = -2 \cdot \csc \frac{2}{3}x$

Period: _____



Directions: Graph each function, showing the asymptotes on the graph. Identify the period, phase shift, and vertical shift of each function.

7. $f(x) = \frac{3}{2} \cdot \sec(x + \pi) - 1$



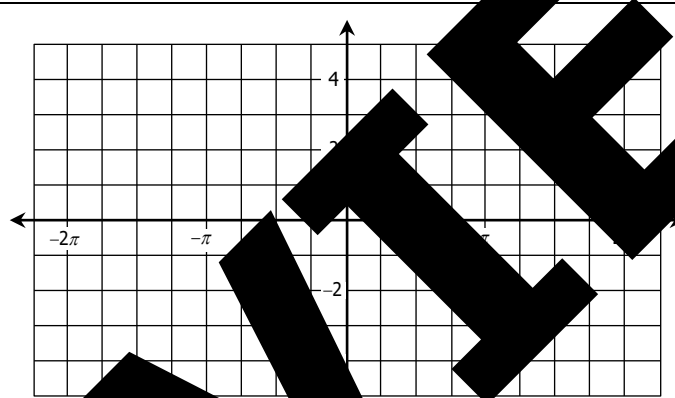
Period:

Phase Shift:

Vertical Shift:

Midline:

8. $f(x) = \cot\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right) + 2$



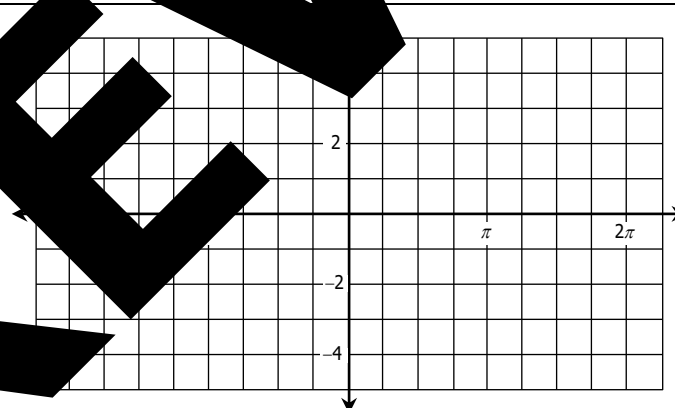
Period:

Phase Shift:

Vertical Shift:

Midline:

9. $f(x) = -\csc\left(x + \frac{\pi}{2}\right) + 4$



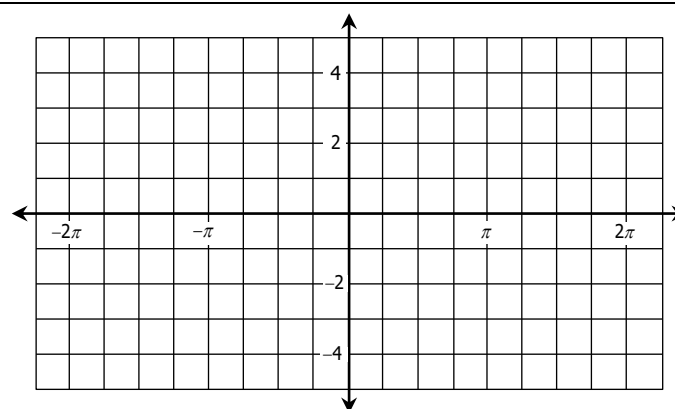
Period:

Phase Shift:

Vertical Shift:

Midline:

$f(x) = \csc\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$



Period:

Phase Shift:

Vertical Shift:

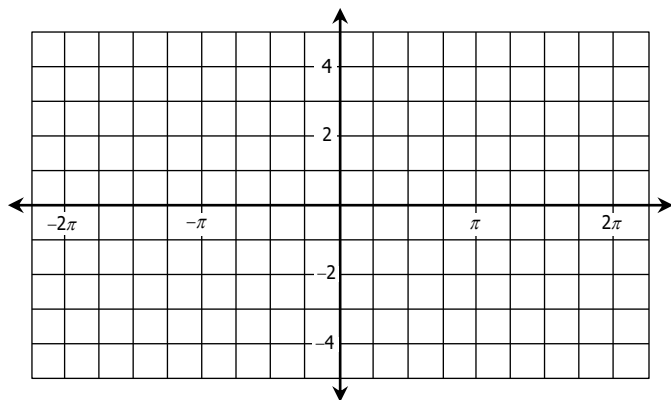
Midline:

GRAPHING TRIGONOMETRIC FUNCTIONS *Review!*

Set I (no shifts):

Graph the function, then identify the amplitude and period.

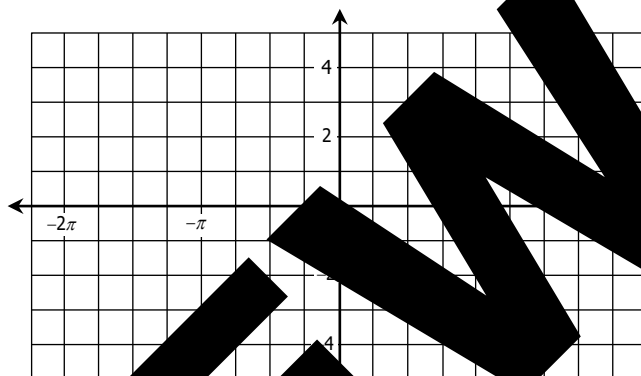
1. $f(x) = 5 \cdot \cos 4x$



Amplitude:

Period:

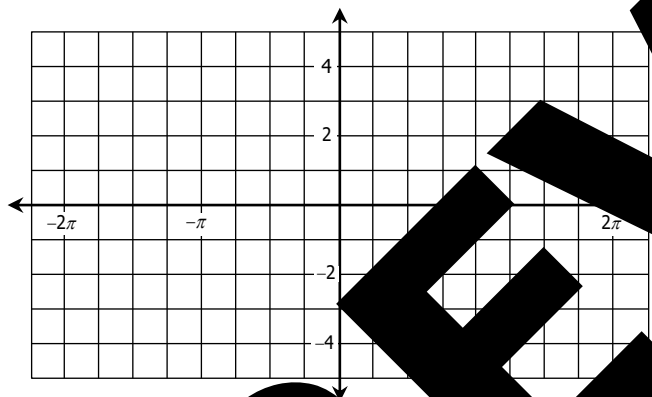
2. $f(x) = \tan \frac{2}{3}x$



Amplitude:

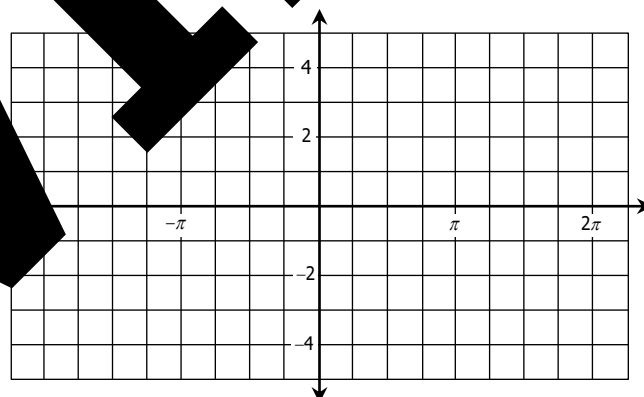
Period:

3. $f(x) = \frac{3}{2} \cdot \csc 2x$



Amplitude:

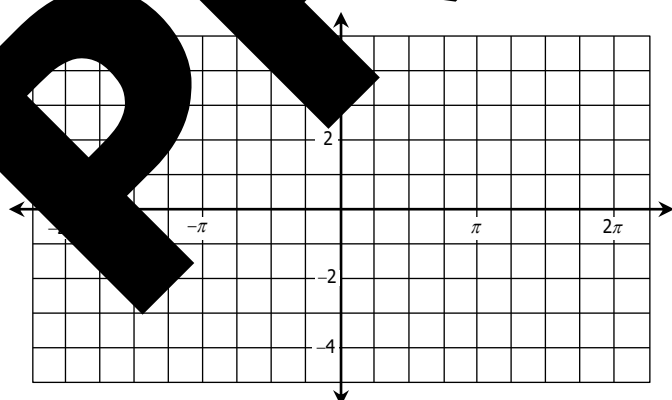
4. $f(x) = -2 \cdot \sec x$



Amplitude:

Period:

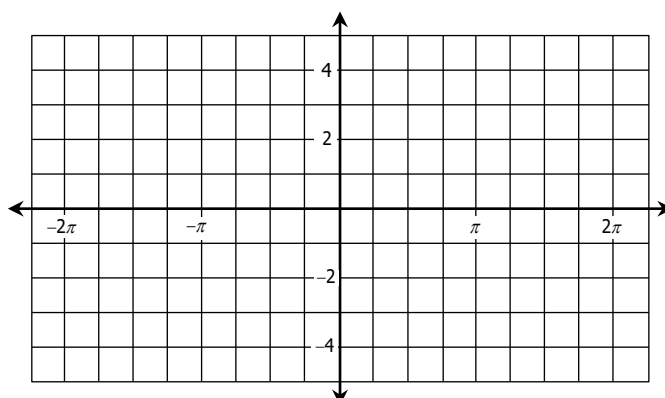
5. $f(x) = \frac{1}{2} \cdot \csc x$



Amplitude:

Period:

6. $f(x) = 3 \cdot \sin \frac{8}{5}x$



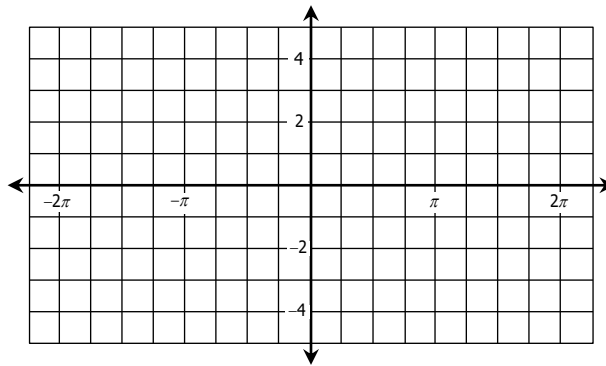
Amplitude:

Period:

Set 2 (with shift):

Graph the function, then identify the characteristics.

7. $f(x) = 3 \cdot \tan\left(x - \frac{\pi}{4}\right) - 3$



Amplitude:

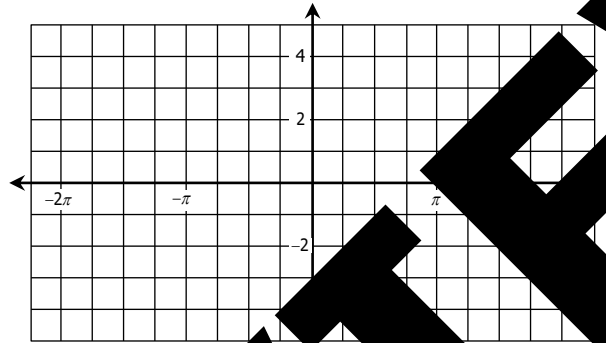
Period:

Phase Shift:

Vertical Shift:

Midline:

8. $f(x) = 2 \cdot \sec\left(\frac{4}{3}(x + \pi)\right)$



Amplitude:

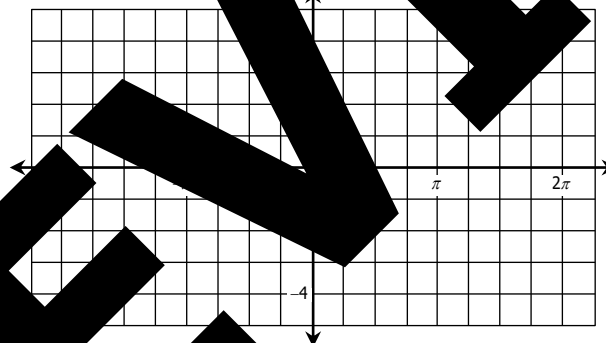
Period:

Phase Shift:

Vertical Shift:

Midline:

9. $f(x) = 2 \cdot \sin\left(x - \frac{3\pi}{2}\right) + 1$



Amplitude:

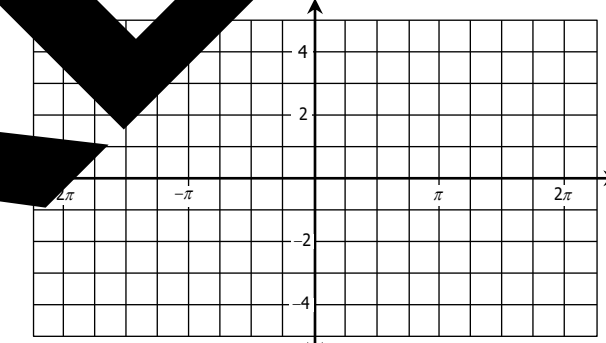
Period:

Phase Shift:

Vertical Shift:

Midline:

10. $f(x) = \csc 2\left(x - \frac{\pi}{4}\right) + 1$



Amplitude:

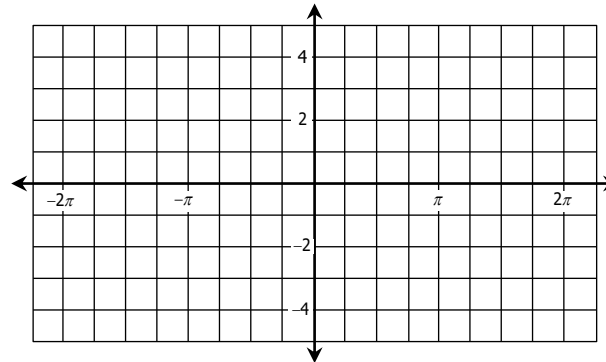
Period:

Phase Shift:

Vertical Shift:

Midline:

11. $f(x) = \cos 4(x + 2\pi) + 1$



Amplitude:

Period:

Phase Shift:

Vertical Shift:

Midline:

Name: _____

Unit 5: Trigonometric Functions



Date: _____ Per: _____

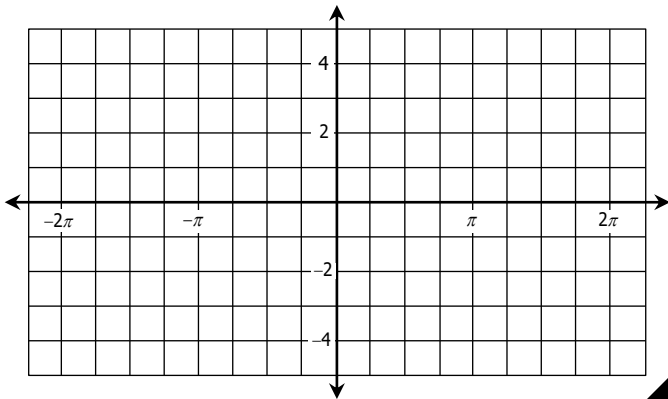
Homework 13: Graphing Trig Functions Review

**** This is a 2-page document! ****

Directions: Identify the amplitude and period, then graph the function.

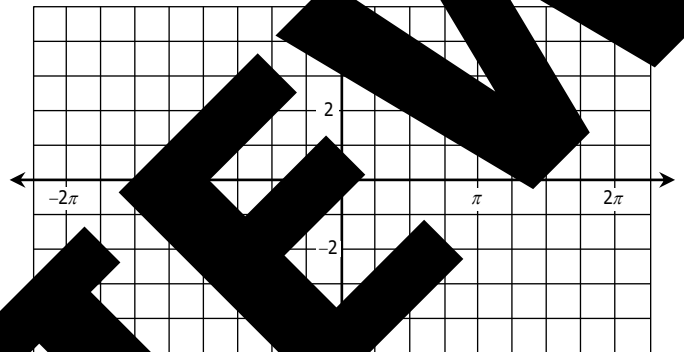
1. $f(x) = 4 \cdot \sin 2x$

Amplitude: _____ Period: _____



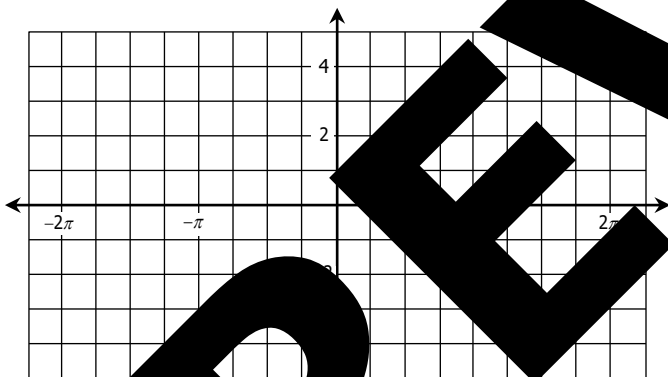
2. $f(x) = -\tan x$

Amplitude: _____ Period: _____



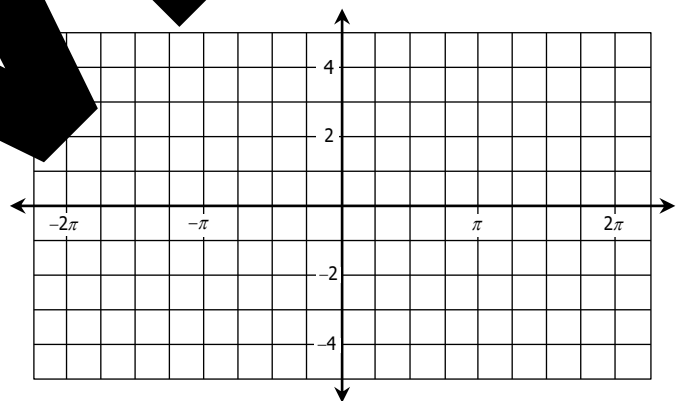
3. $f(x) = -\frac{3}{2} \cdot \cos 4x$

Amplitude: _____ Period: _____



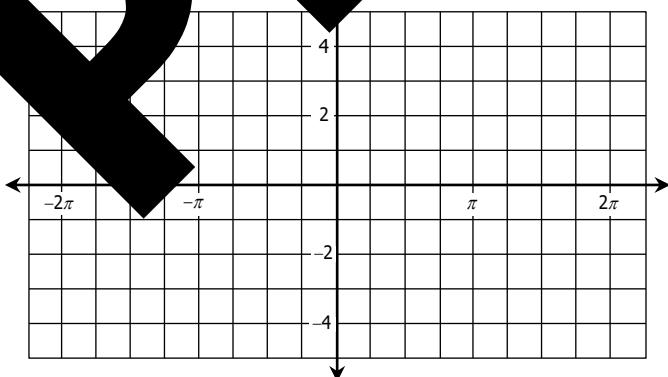
4. $f(x) = 3 \cdot \sec \frac{1}{2}x$

Amplitude: _____ Period: _____



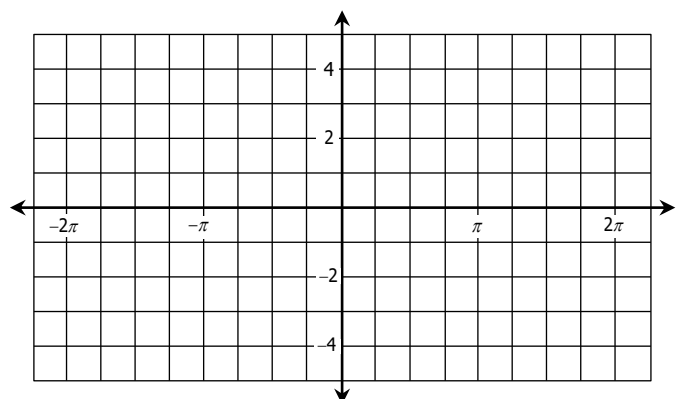
5. $f(x) = 5 \cdot \sec \frac{1}{3}x$

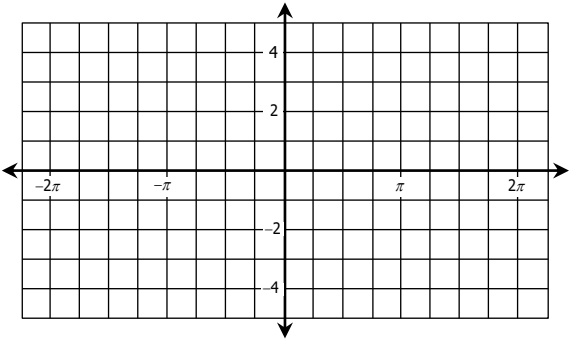
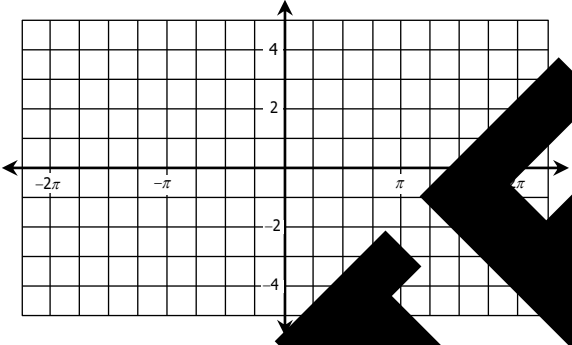
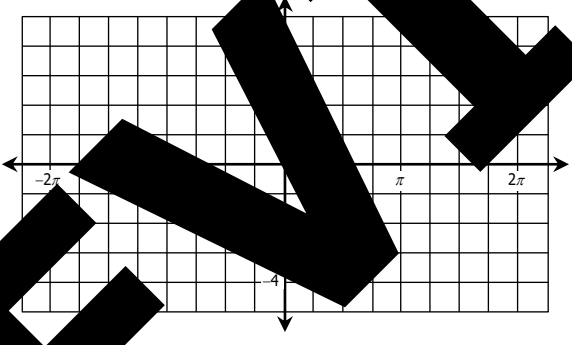
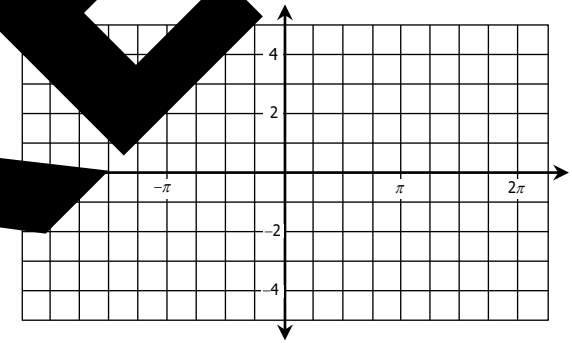
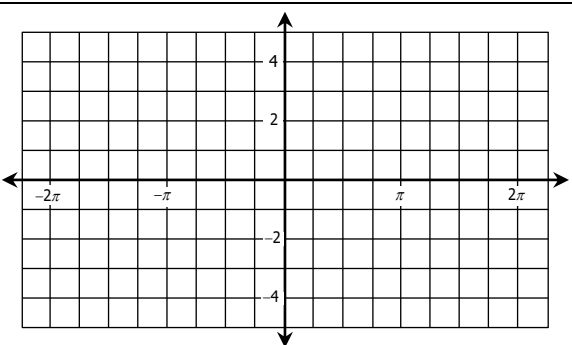
Period: _____



6. $f(x) = 2 \cdot \cos \frac{1}{2}x$

Amplitude: _____ Period: _____



7. $f(x) = 4 \cdot \cos(x + \pi)$		Amplitude: Period: Phase Shift: Vertical Shift: Midline:
8. $f(x) = 3 \cdot \sec\left(x - \frac{\pi}{4}\right) + 1$		Amplitude: Period: Phase Shift: Vertical Shift: Midline:
9. $f(x) = 2 \cdot \cot 2\left(x + \frac{\pi}{4}\right)$		Amplitude: Period: Phase Shift: Vertical Shift: Midline:
10. $f(x) = 4 \cdot \tan \frac{1}{2}\left(x - \frac{3\pi}{4}\right)$		Amplitude: Period: Phase Shift: Vertical Shift: Midline:
11. $f(x) = \sin \frac{2}{3}\left(x + \frac{\pi}{2}\right) - 1$		Amplitude: Period: Phase Shift: Vertical Shift: Midline:

Name: _____

Pre-Calculus

Date: _____ Per: _____

Unit 5: Trigonometric Functions

Quiz 5-4: Graphing Trigonometric Functions

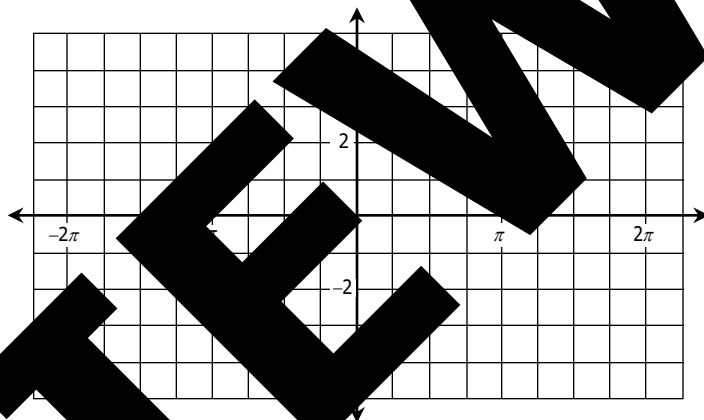
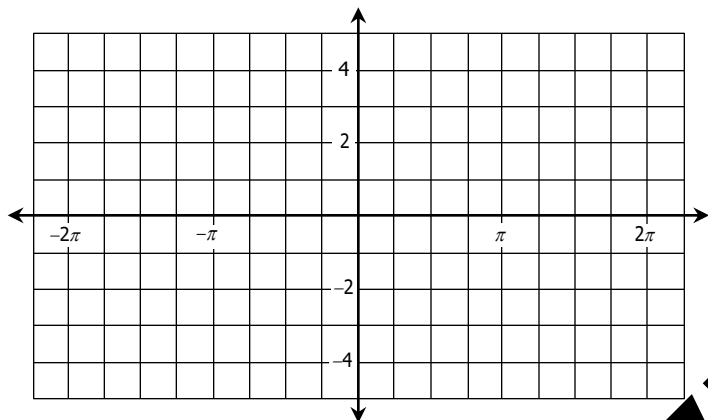
Directions: Give the amplitude and period for each function. Then graph.

1. $f(x) = 5 \cdot \sin 2x$

Amp: _____ Per: _____

2. $f(x) = \cot \frac{2}{3}x$

Amp: _____ Per: _____

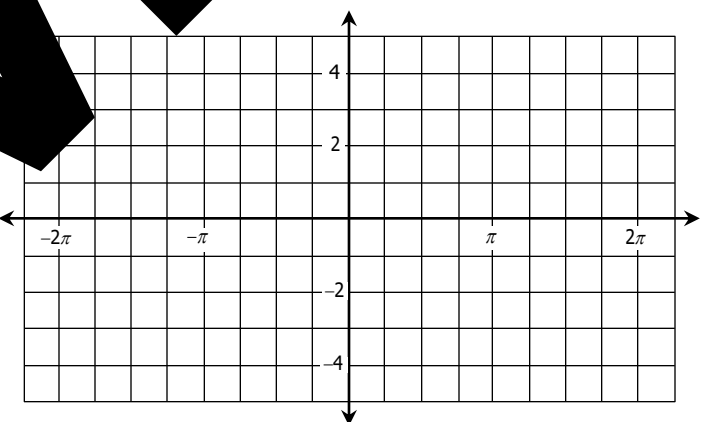
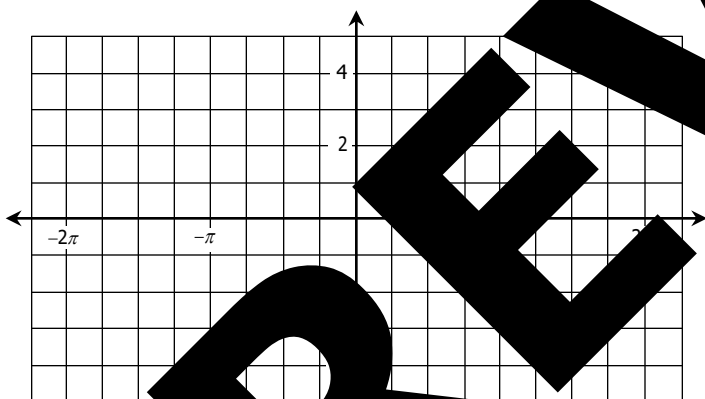


3. $f(x) = \frac{1}{2} \cdot \sec x$

Amp: _____ Per: _____

4. $f(x) = \frac{1}{2} \cdot \csc x$

Amp: _____ Per: _____

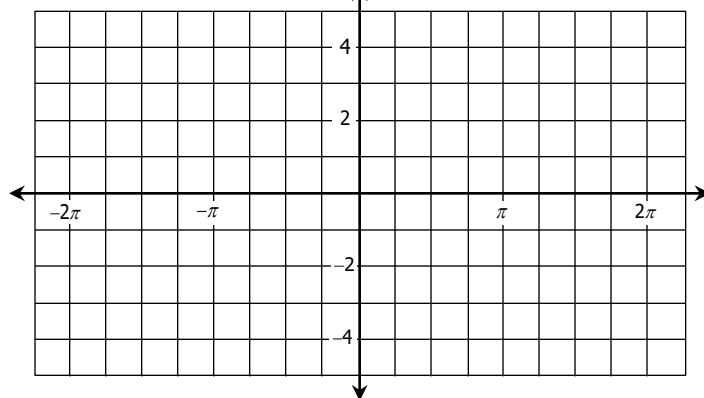
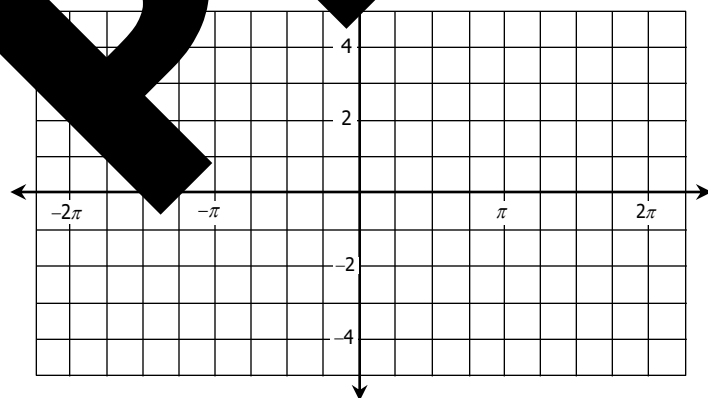


5. $f(x) = \tan \frac{1}{3}x$

Amp: _____ Per: _____

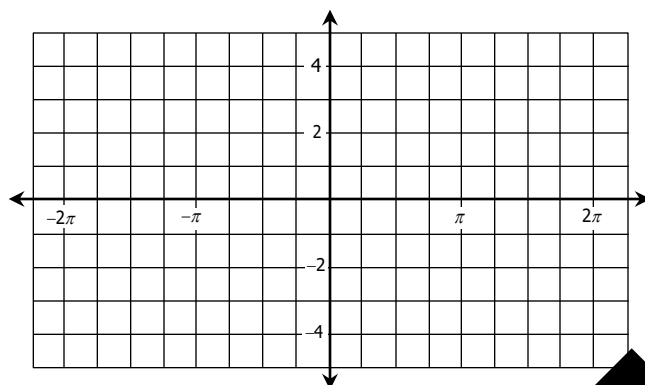
6. $f(x) = -4 \cdot \cos \frac{1}{2}x$

Amp: _____ Per: _____



Directions: Identify the amplitude, period, translations, and midline. Then graph.

7. $f(x) = \tan 2\left(x + \frac{3\pi}{2}\right) + 3$



Amplitude: _____

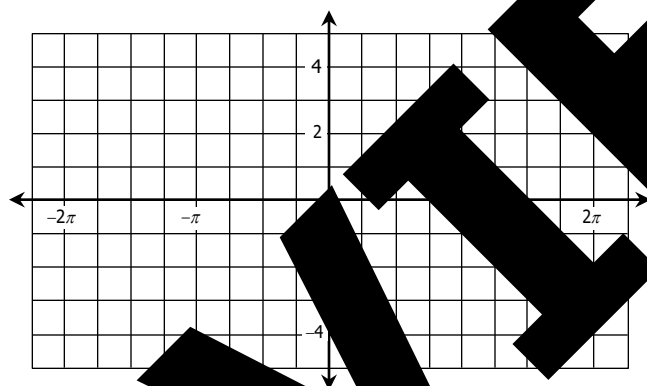
Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

8. $f(x) = 4 \cdot \sin \frac{2}{3}(x - 2\pi) + 1$



Amplitude: _____

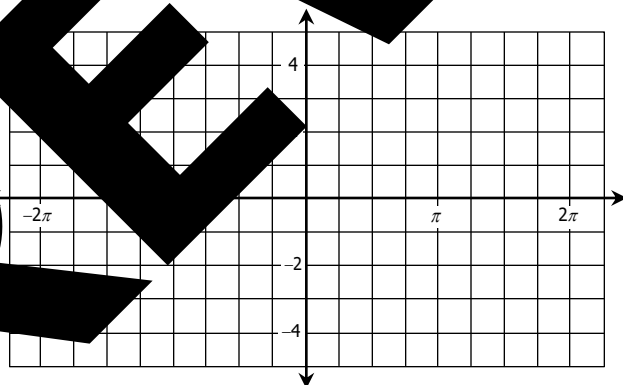
Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

9. $f(x) = \cos 4(x - \pi) - 4$



Amplitude: _____

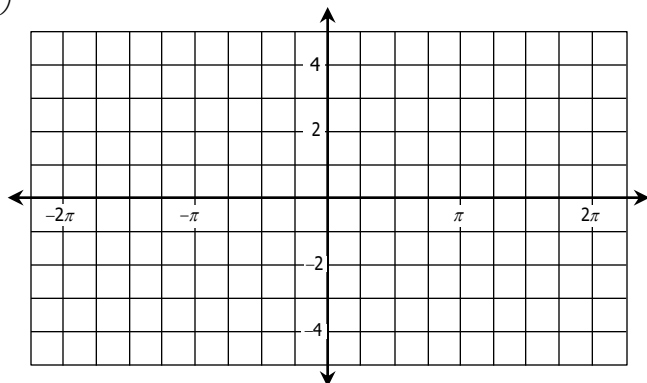
Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

10. $f(x) = -\csc\left(x + \frac{\pi}{4}\right) - 2$



Amplitude: _____

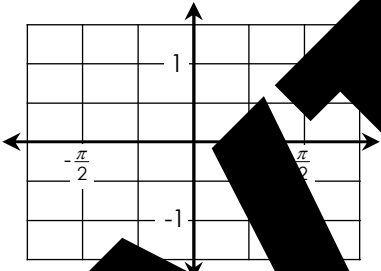
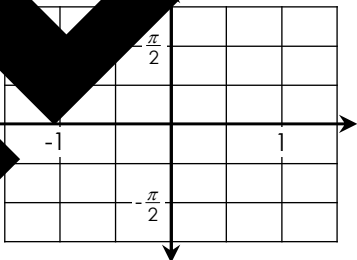
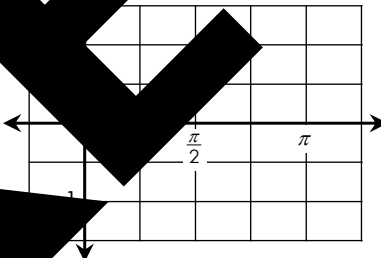
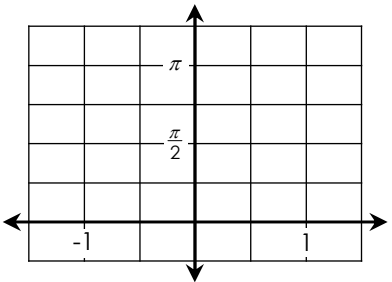
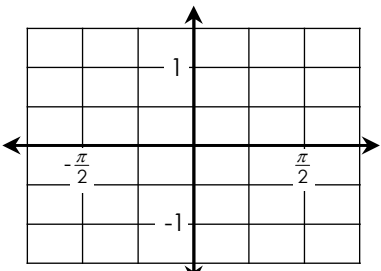
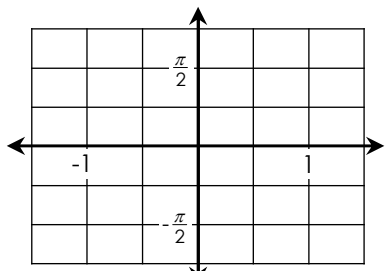
Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
INVERSE <i>Trigonometric</i> FUNCTIONS	A function has an inverse if and only if it is a one-to-one function. (This means that it must pass the horizontal line test to have an inverse.) However, if the domain is restricted to a certain interval, the sine, cosine, and tangent functions have inverse functions.	
	When graphed on a restricted interval: ➤ the inverse of the sine function is the arcsine function ➤ the inverse of the cosine function is the arccosine function ➤ the inverse of the tangent function is the arctangent function	
ARCSINE <i>Function</i>	$f(x) = \sin x; -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	
		
ARCCOSINE <i>Function</i>	$f(x) = \cos x; 0 \leq x \leq \pi$	$f(x) = \arccos x$ or $f(x) = \cos^{-1} x$
		
ARCTANGENT <i>Function</i>	$f(x) = \tan x; -\frac{\pi}{2} < x < \frac{\pi}{2}$	$f(x) = \arctan x$ or $f(x) = \tan^{-1} x$
		

EVALUATING INVERSE FUNCTIONS

Use the unit circle to help you evaluate inverse trigonometric functions! However, remember that the domain and range are restricted on trigonometric functions.

Directions: Find the exact values of each expression, if it exists.

1. $\arcsin 1$

2. $\sin^{-1} \frac{\sqrt{3}}{2}$

3. $\arcsin(-2)$

4. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

5. $\cos^{-1} 3$

6. $\arctan(-1)$

7. $\arctan \frac{\sqrt{3}}{3}$

8. $\arctan 6$

9. $\tan^{-1} \sqrt{3}$

APPLICATIONS

10. A ship left Port A and sailed 120 miles due east to Port B. From Port B, the ship sailed 120 miles northwest and stopped at a point directly north of Port A. Find the bearing from Port A to the current location of the ship.

The angle of elevation from a football on a football field to the top of a goal post depends on d , the distance from the ball to the base of the goal post.

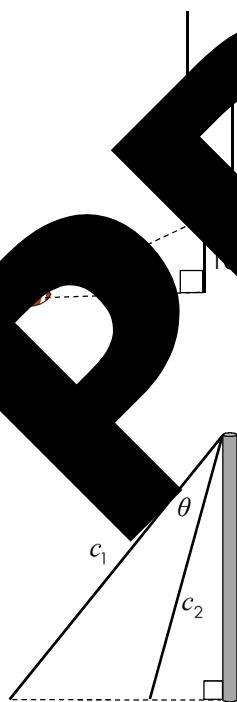
a) Write θ as a function of d . _____

b) Using your graphing calculator, find θ when d is 75 feet. Round to the nearest tenth of a degree.

12. Two cables are secured from the top of a 50-foot tall utility pole to the ground. The lengths of the cables are given as c_1 and c_2 .

a) Write θ as a function of c_1 and c_2 . _____

b) Using your graphing calculator, find θ if the cables are 120 feet and 65 feet long.



Name:

Date:

Topic:

Class:

Main Ideas/Questions

Notes/Examples

Transformations
of **INVERSE TRIG
FUNCTIONS**

One strategy to graph transformations of inverse function is to find the inverse function, create a table of values, then invert the values to graph.

Example 1: Graph $f(x) = \arcsin(2x)$

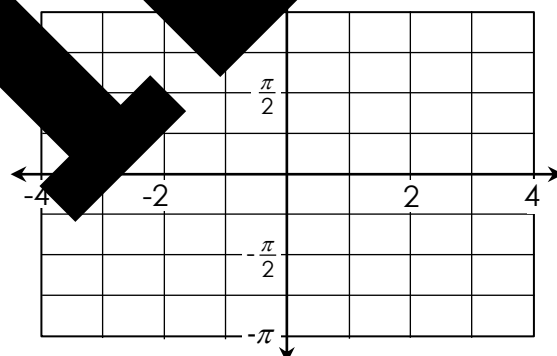
Step 1:

Find the inverse of the function.

Step 2: Make a table of values.

x					
$f^{-1}(x)$					

Step 3: Graph the inverted points.



Note: This shifts the range down π , therefore the range is now $[-\pi, 0]$.

Example 2: Graph $f(x) = \cos^{-1}x - \pi$

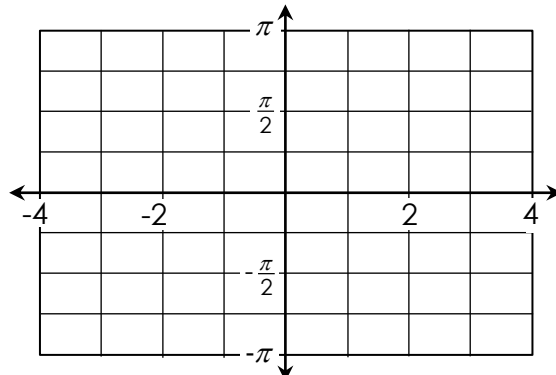
Step 1:

Find the inverse of the function.

Step 2: Make a table of values.

x					
$f^{-1}(x)$					

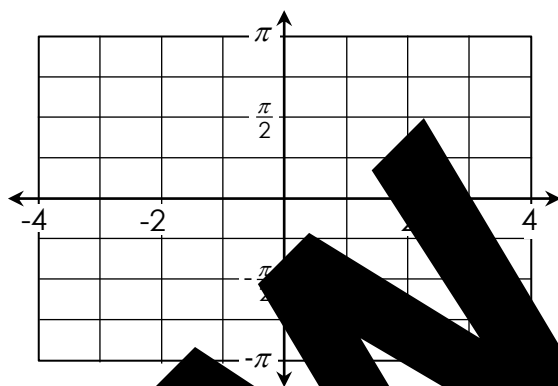
Step 3: Graph the inverted points.



Example 3:

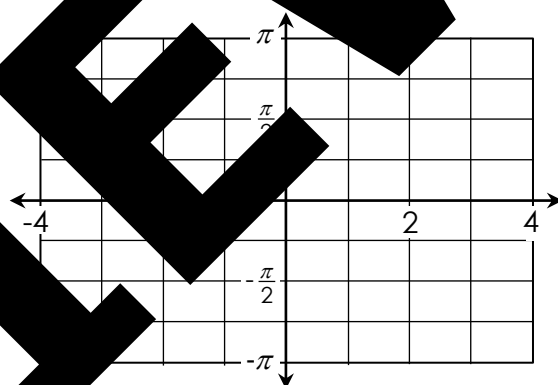
$$f(x) = \arctan(x+2)$$

x	$f^{-1}(x)$

**Example 4:**

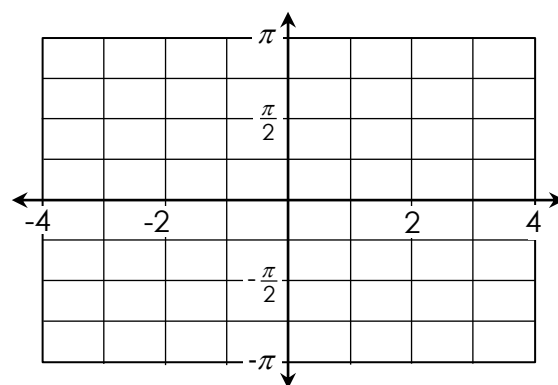
$$f(x) = \cos^{-1} \frac{1}{4}x$$

x	$f^{-1}(x)$

**Example 5:**

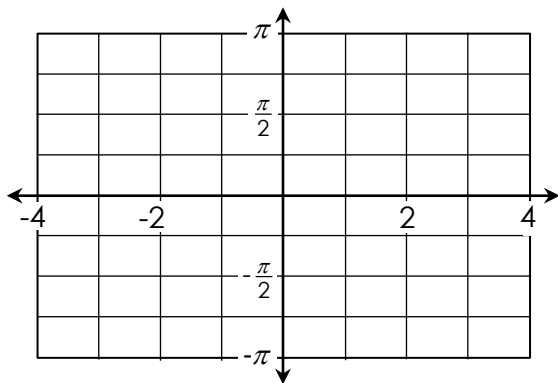
$$f(x) = \arcsin x + \frac{\pi}{2}$$

x	$f^{-1}(x)$

**Example 6:**

$$f(x) = \tan^{-1}(x-1)$$

x	$f^{-1}(x)$



Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 14: Inverse Trigonometric Functions

** This is a 2-page document! **

Directions: Find the exact values of each expression, if it exists.

1. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

2. $\tan^{-1} 1$

3. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

4. $\arcsin\left(\frac{1}{2}\right)$

5. $\arccos 2$

6. $\arctan(-\sqrt{3})$

7. $\sin^{-1} \frac{1}{2}$

8. $\arctan\left(\frac{1}{3}\right)$

9. Molly is looking up at the the St. Louis Arch, which stands 630 feet high. The angle of elevation, q , from her line of sight to the top of the monument depends on the distance, d , she is from the base. If Molly's eyes are 5 feet 3 inches from the ground, write a function for q in terms of d .

10. A boat leaves the dock and travels 172 m before dropping the anchor. The angle of depression depends on the straightline distance between the end of the dock and the anchor.

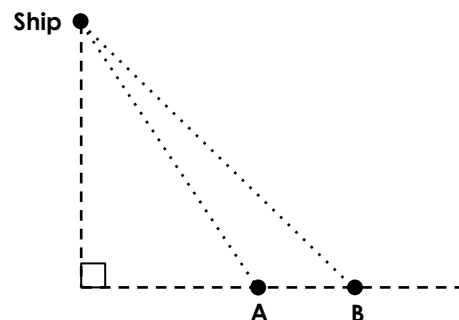
a) Write q as a function of d .

b) Using your calculator, find q when d is 205 meters. Round to the nearest tenth of a degree.

The captain on a ship picks up two shipwrecks, A and B, 2.5 miles beneath the surface. The distance between the shipwrecks is 0.6 miles. The angle of depression, q , to Shipwreck B depends on the horizontal distance, d , from the ship to Shipwreck A.

a) Write a function for q in terms of d .

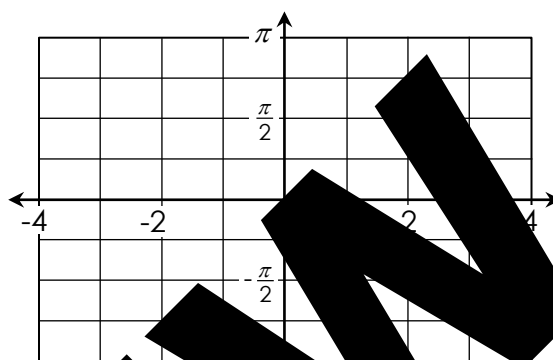
b) Find q if d is 1.4 miles, rounding to the nearest tenth.



Directions: Graph each of the following.

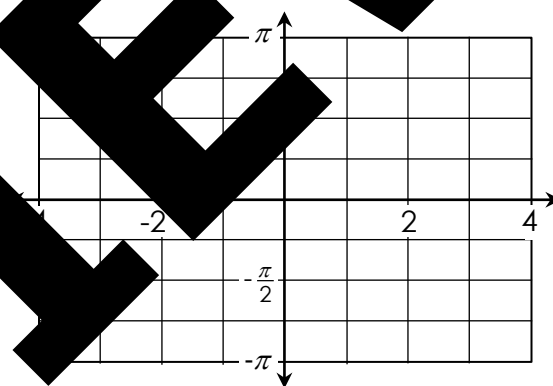
12. $f(x) = \arctan 2x$

x	$f^{-1}(x)$



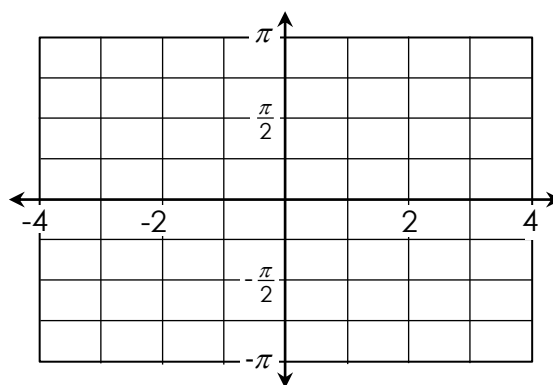
13. $f(x) = \cos^{-1} x - \frac{3\pi}{4}$

x	$f^{-1}(x)$



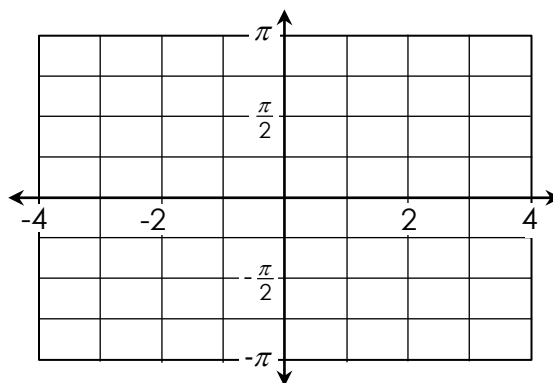
14. $f(x) = \arcsin(x - 3)$

x	$f^{-1}(x)$



$f(x) = \arcsin\left(\frac{1}{2}(x+1)\right) + \frac{\pi}{2}$

x	$f^{-1}(x)$



Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
COMPOSITIONS of Trigonometric Functions Note: If x is an angle in the domain of f, then $f^{-1}(f(x)) = x$. If y is a value in the range of f, then $f(f^{-1}(y)) = y$.	Recall: Given function f and its inverse f^{-1}, if x is in the domain of both functions, then $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$.
	The properties of compositions of trigonometric functions are summarized below:
	If x is in the domain of $\sin^{-1}$, then $\sin(\sin^{-1} x) = x$. If x is in the domain of $\cos^{-1}$, then $\cos(\cos^{-1} x) = x$.
	If x is in the domain of $\tan^{-1}$, then $\tan(\tan^{-1} x) = x$. If x is in the domain of $\sec^{-1}$, then $\sec(\sec^{-1} x) = x$.
	Directions: Find the exact value of each expression, if it exists.
	1. $\sin\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]$ 2. $\cos\left[\cos^{-1}\left(\frac{2}{3}\right)\right]$ 3. $\cos\left(\cos^{-1}\frac{3}{2}\right)$
	4. $\tan\left[\tan^{-1}\left(-\frac{1}{2}\right)\right]$ 5. $\sin\left(\tan^{-1}\frac{1}{4}\right)$ 6. $\sin\left[\arcsin\left(-\frac{7}{6}\right)\right]$
	7. $\sin\left(\tan^{-1}\frac{1}{4}\right)$ 8. $\cos^{-1}\left(\cos\frac{5\pi}{6}\right)$
	9. $\tan\left[\tan^{-1}\left(-\frac{\pi}{2}\right)\right]$ 10. $\sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right]$
	11. $\arcsin\left(\sin\frac{3\pi}{4}\right)$ 12. $\tan^{-1}(\tan \pi)$
	13. $\sin^{-1}\left(\sin\frac{11\pi}{6}\right)$ 14. $\arccos\left[\cos\left(-\frac{4\pi}{3}\right)\right]$

MIXING FUNCTIONS SET 1: Unit Circle Values	Find the exact value of each expression, if it exists. Work inside out!	
	15. $\cos^{-1}\left(\sin \frac{\pi}{6}\right)$	16. $\arctan (\cos 0)$
	17. $\sin^{-1}\left[\tan \left(-\frac{\pi}{4}\right)\right]$	18. $\arcsin \left(\cos \frac{5\pi}{6}\right)$
	19. $\cos^{-1}\left(\tan \frac{\pi}{4}\right)$	20. $\arctan \left[\sin \left(-\frac{\pi}{2}\right)\right]$
	21. $\cos \left(\sin^{-1} \frac{1}{2}\right)$	22. $\sin \left(\cos^{-1} \frac{1}{2}\right)$
	23. $\sin (\arccos \frac{1}{2})$	24. $\cos \left(\tan^{-1} \sqrt{3}\right)$
SET 2: Non-Unit Circle Values	• Determine the "inside" angle θ. • Determine which quadrant θ must lie. • Draw a right triangle and find the missing side. • Evaluate the outside trigonometric function. (Keep the quadrant in mind.)	
	25. $\sin \left(\cos^{-1} \frac{8}{17}\right)$ 26. $\sin \left[\arctan \left(-\frac{12}{5}\right)\right]$	

Name: _____

Unit 5: Trigonometric Functions



Date: _____ Per: _____

Homework 15: Compositions of Trigonometric Functions

Directions: Find the exact value of each expression, if it exists.

1. $\cos\left(\arccos \frac{4}{5}\right)$

2. $\tan\left(\tan^{-1} \frac{7}{3}\right)$

3. $\sin\left[\sin^{-1}\left(-\frac{5}{7}\right)\right]$

4. $\tan^{-1}\left(\tan \frac{\pi}{3}\right)$

5. $\cos^{-1}(\cos \pi)$

6. $\cos\left[\sin^{-1}\left(\frac{3}{4}\right)\right]$

7. $\arctan\left(\tan \frac{5\pi}{6}\right)$

8. $\arccos\left(\cos \frac{3\pi}{2}\right)$

9. $\sin\left[\sin^{-1}\left(-\frac{5\pi}{3}\right)\right]$

10. $\cos\left(\arcsin \frac{\sqrt{3}}{2}\right)$

11. $\tan\left(\arccos \frac{\sqrt{2}}{2}\right)$

12. $\cos^{-1}\left[\sin\left(-\frac{11\pi}{6}\right)\right]$

13. $\sin\left(\tan^{-1} \frac{\sqrt{3}}{3}\right)$

14. $\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$

15. $\sin[\arctan(-1)]$

16. $\sin\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$

17. $\cos\left(\tan^{-1} \frac{3}{2}\right)$

Unit 5 Test Study Guide (Trigonometric Functions)

Name: _____

Date: _____ Per: _____

Topic 1: Angles & Angle Measures

Directions: Change the degrees to radians, and the radians to degrees.

1. 310°

2. -155°

3. $\frac{7\pi}{3}$

4. $-\frac{4\pi}{3}$

Directions: State the quadrant in which the terminal side lies, then, give two coterminal angles, one positive and one negative, for each angle.

5. -285°

6. 2°

Directions: Write each angle measure in Degree-Minute-Second (DMS) form.

7. 56.875°

8. -207.143°

Directions: Write each angle measure in decimal degree form.

9. $-184^\circ 16' 40''$

10. $78^\circ 37' 3''$

Topic 2: Arc Lengths, Area of Sectors, & Circular Motion

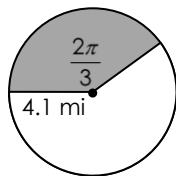
Directions: Find the length of the intercepted arc. Round to the nearest tenth.

11. $\frac{5\pi}{6}$; $r = 3.5$ ft

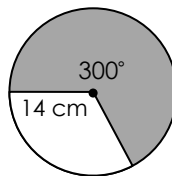
12. 120° ; $r = 6$ in

Directions: Find the area of each sector. Round to the nearest tenth.

13.



14.



For questions 15 and 16: An 18-inch bicycle tire is making 140 revolutions per minute.

15. Find the linear speed in miles per hour.

16. Find the angular speed of the bicycle tire in radians per minute.

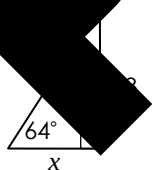
For questions 17 and 18: A CD with a radius of 6 centimeters rotates at a speed of 237 revolutions per minute.

17. Find the linear speed in meters per second.

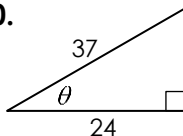
18. Find the angular speed in radians per minute.

Topic 3: Trigonometric Functions & Solving Right Triangles

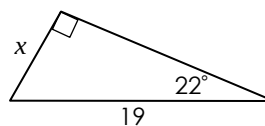
Directions: Find the missing side or angle measure. Round answers to the nearest tenth.



20.



21.



22. A 30-foot ladder is leaning against an office building. If the base of the ladder is 17 feet from the base of the building, find the angle of elevation.

23. Sharon is bird watching and spots an eagle in a tree. When standing 40 meters from the base of the tree, the angle of elevation from the point in which she is standing is 37° . After moving closer to the tree, Sharon now measures a $51^\circ 42'$ angle of elevation. Determine the distance Sharon traveled between viewings.

Topic 4: Trigonometric Functions for Any Angle and the Unit Circle

24. $A(2, -11)$ is a point on the terminal side of a standard form angle θ . Find the exact values of the trigonometric functions of θ .

$\sin \theta =$	$\csc \theta =$
$\cos \theta =$	$\sec \theta =$
$\tan \theta =$	$\cot \theta =$

25. If $\cos \theta = \frac{6}{5}$ and $\theta < 0$, find the exact values of each of the trigonometric functions of θ .

$\sin \theta =$	$\csc \theta =$
$\cos \theta =$	$\sec \theta =$
$\tan \theta =$	$\cot \theta =$

Directions: Find the reference angle for each of the following.

26. 164°

27. -461°

28. $-\frac{3\pi}{4}$

29. $\frac{13\pi}{9}$

Directions: Give the exact value of each trigonometric function.

30. $\sin\left(-\frac{4\pi}{3}\right)$

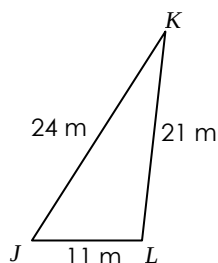
31. $\sec\left(-\frac{7\pi}{6}\right)$

32. $\cos\frac{7\pi}{4}$

33. $\cot\frac{\pi}{6}$

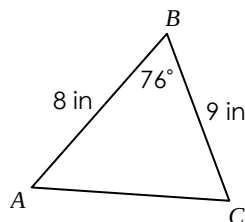
Topic 5: Law of Sines & Law of Cosines

34. Solve the triangle below. Round all answers to the nearest tenth.



$m\angle J =$
$m\angle K =$
$m\angle L =$

35. Find AC.

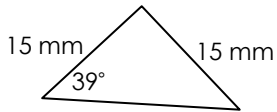


36. In triangle DEF, $m\angle D = 42^\circ$, $m\angle E = 17^\circ$, $DE = 17$ m, $EF = 16$ m, find DF .

37. The log ride at an amusement park has a single hill with an incline of 127 feet and a decline of 93 feet. If the total horizontal distance between the start of the incline and end of the decline is 120 feet, find the total height of the ride.

38. Two towers at a national park spot a forest fire. Tower A spots the fire at a bearing of N 38° W and Tower B spots the fire at a bearing of N 48° E. If Tower A is 729 meters directly east of Tower B, find the distance from each tower to the fire.

39. Find the area of the triangle below.



40. A triangular plaque has side lengths of 8 inches, 13 inches and 15 inches. Use Heron's Formula to find the area of the plaque.

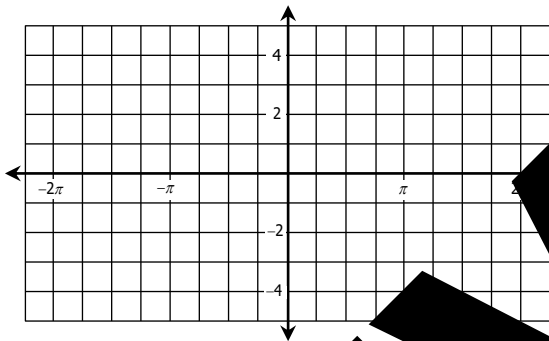
Topic 6: Graphing Trigonometric Functions

Directions: Identify the amplitude and period of each function, then graph.

41. $f(x) = -\sin 4x$

Amplitude: _____

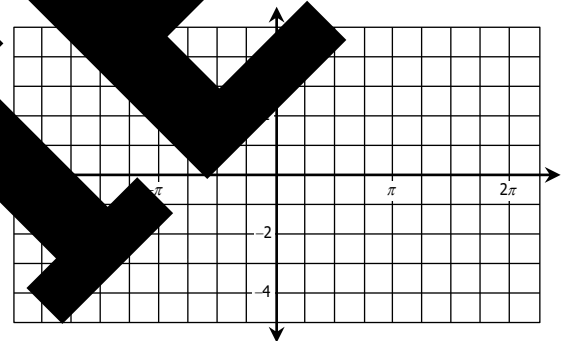
Period: _____



42. $f(x) = 2 \sin \frac{\pi}{3} x$

Amplitude: _____

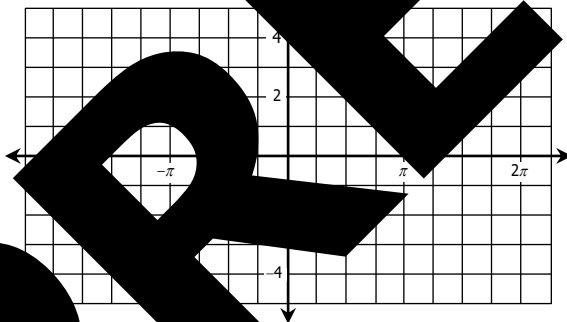
Period: _____



43. $f(x) = \frac{3}{2} \cdot \sec \frac{8}{3} x$

Amplitude: _____

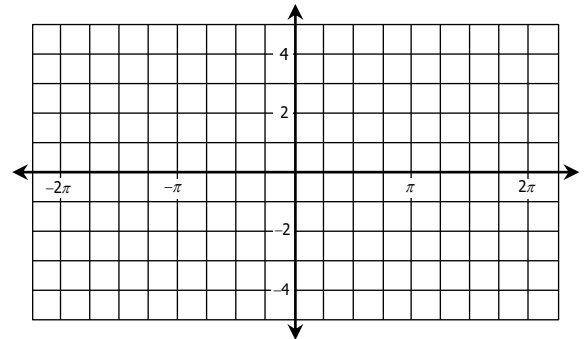
Period: _____



44. $f(x) = 3 \cdot \csc 2x$

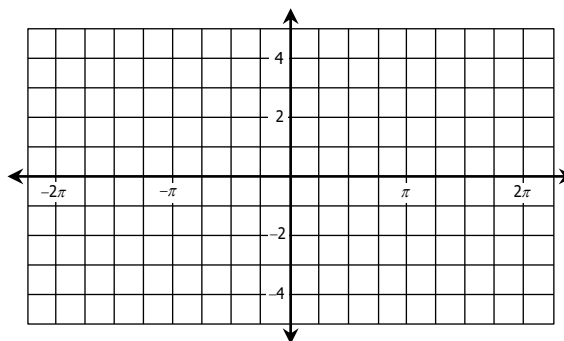
Amplitude: _____

Period: _____



Directions: Identify the amplitude, period, translations, and midline of each function. Then graph.

45. $f(x) = \frac{1}{2} \cdot \cot(x - \pi) + 3$



Amplitude: _____

Period: _____

Phase Shift: _____

Vertical Shift: _____

Midline: _____

46. $f(x) = \frac{5}{2} \cdot \cos \frac{2}{3} \left(x + \frac{3\pi}{4} \right) - 1$

Amplitude:

Period:

Phase Shift:

Vertical Shift:

Midline:

Topic 7: Inverse Trigonometric Functions & Compositions of Trigonometric Functions

Directions: Give the exact value, if it exists.

47. $\sin^{-1} \frac{\sqrt{2}}{2}$	48. $\arctan(-\sqrt{3})$	49. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
50. $\cos\left[\cos^{-1}\left(-\frac{2}{7}\right)\right]$	51. $\tan(\tan^{-1} 3)$	52. $\sin\left[\cos^{-1}\left(\frac{3}{4}\right)\right]$
53. $\arctan\left[\tan\left(-\frac{\pi}{6}\right)\right]$	54. $\cos^{-1}(\cos \pi)$	55. $\arcsin\left[\sin\left(-\frac{3\pi}{2}\right)\right]$
56. $\arccos\left[\cos\left(\frac{11\pi}{6}\right)\right]$	57. $\sin^{-1}\left(\sin^{-\frac{2}{3}}\right)$	58. $\tan^{-1}\left[\tan\left(-\frac{5\pi}{4}\right)\right]$
59. $\sin\left(\cos^{-1} \sqrt{\frac{5}{13}}\right)$	60. $\tan^{-1}(\tan^{-1} 1)$	61. $\tan[\arcsin(-1)]$
62. $\sin\left(\cos^{-1} \frac{\pi}{3}\right)$	63. $\arctan(\sin 2\pi)$	64. $\arccos\left[\sin\left(-\frac{3\pi}{4}\right)\right]$
65. $\sin\left[\tan^{-1}\left(-\frac{24}{7}\right)\right]$	66. $\tan\left(\cos^{-1} \frac{3\sqrt{5}}{11}\right)$	

Name: _____

Date: _____ Per: _____

Unit 5 Test

Trigonometric Functions

1. Write 252° in radians.

2. Write $-\frac{2\pi}{9}$ in degrees.

3. When graphed in standard form, the terminal side of an angle measuring $\frac{11\pi}{4}$ lies in which quadrant?

- A. I
- B. II
- C. III
- D. IV

4. Which angles are coterminal with 150° ? Check all that apply.

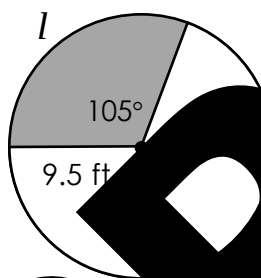
☐ $\frac{5\pi}{6}$	☐ $\frac{5\pi}{8}$	☐ $\frac{11\pi}{6}$
☐ $\frac{\pi}{6}$	☐ $\frac{11\pi}{8}$	☐ $\frac{\pi}{6}$

5. Write 61.595° in degree-minute-second form.

6. Write $230^\circ 27'$ in decimal degrees. Round to the nearest thousandth.

Use the circle below for questions 7 and 8.

(Round each answer to the nearest tenth.)



7. Find l , the length of the arc of the shaded sector.

$l =$

8. Find A , the area of the shaded sector.

$A =$

9. A school bus tire with a diameter of 42 inches is rotating at a rate of 250 revolutions per minute.

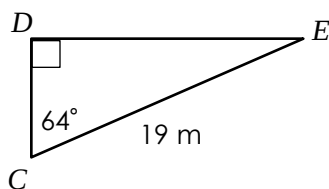
Find the linear speed of the bus in miles per hour. Round to the nearest tenth.

- A. 25.3 mph
- B. 26.5 mph
- C. 28.8 mph
- D. 31.2 mph

10. Find the angular speed of the tire in radians per minute. Round to the nearest tenth.

- A. 1,570.8 radians/minute
- B. 942.5 radians/minute
- C. 1,292.6 radians/minute
- D. 1,804.1 radians/minute

11. Find CD to the nearest tenth of a meter.



$CD =$

12. Find $m\angle W$ to the nearest tenth of a degree.



$m\angle W =$

13. A fireman spots a stranded woman standing in front of a 200-foot tall office building. From the point where the fireman is standing, the angle of elevation to the stranded woman is $42^\circ 33'$ and to the top of the building is $58^\circ 6'$. How far from the ground is the stranded woman?

- A. 108.8 feet
- B. 114.5 feet
- C. 121.1 feet
- D. 124.5 feet

If $M(1, 0)$ is a point on the terminal side of θ in standard form, find the exact values of the trigonometric functions of θ .

$\sin \theta =$	$\csc \theta =$
$\cos \theta =$	$\sec \theta =$
$\tan \theta =$	$\cot \theta =$

15. If $\cot \theta = -\frac{24}{7}$, and $\cos \theta < 0$, name the quadrant in which θ lies, then find the exact value of $\csc \theta$.

Quadrant:

$\csc \theta =$

16. Give the reference angle of $\frac{8\pi}{5}$ in radians.

- A. $\frac{\pi}{5}$ C. $\frac{\pi}{10}$
B. $\frac{2\pi}{5}$ D. $\frac{3\pi}{10}$

For questions 17-20, give the exact value of each trigonometric function.

17. $\csc \frac{5\pi}{4}$

- A. $-\frac{2\sqrt{3}}{3}$ C. $-\sqrt{2}$
B. $\frac{2\sqrt{3}}{3}$ D. $\sqrt{2}$

18. $\tan \frac{7\pi}{6}$

- A. $\frac{\sqrt{3}}{3}$ C. $\frac{\sqrt{3}}{3}$
B. $-\frac{\sqrt{3}}{3}$ D. $-\sqrt{3}$

19. $\cos\left(-\frac{2\pi}{3}\right)$

- A. $\frac{\sqrt{3}}{2}$ C. $\frac{1}{2}$
B. $-\frac{\sqrt{3}}{2}$ D. $-\frac{1}{2}$

20. $\sec\left(-\frac{\pi}{6}\right)$

- A. $-\frac{2\sqrt{3}}{3}$ C. $-\frac{\sqrt{3}}{2}$
B. $\frac{2\sqrt{3}}{3}$ D. $\frac{\sqrt{3}}{2}$

21. Which functions have a value of 2? Check all that apply.

☐

$\tan\left(\frac{\pi}{6}\right)$

☐

$\cos \pi$

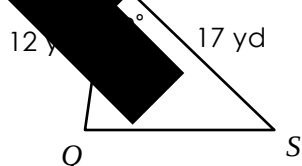
☐

$\sec \frac{3\pi}{2}$

☐

$\cot\left(-\frac{3\pi}{4}\right)$

Solve the triangle below. Round all answers to the nearest tenth.



$m\angle Q =$

$m\angle S =$

$QS =$

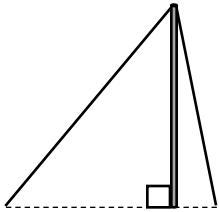
23. In $\triangle ABC$, $m\angle C = 73^\circ$, $AB = 20$ km, and $AC = 29$ km. Find $m\angle B$.

- A. 41.3°
- B. 41.3° or 138.7°
- C. 65.7°
- D. Not possible

24. In $\triangle ABC$, $m\angle B = 82^\circ$, $AC = 34$ mm, and $BC = 14$ mm. Find $m\angle A$.

- A. 73.9°
- B. 41.3° or 155.8°
- C. 24.1°
- D. Not possible

25. Two cables, with lengths of 40 feet and 35 feet, are tied from the top of a pole to stakes in the ground. If the stakes are 30 feet apart, find the height of the pole.



- A. 30.6 ft
- B. 33.9 ft
- C. 36.2 ft
- D. 39.5 ft

26. Philadelphia, Pennsylvania is 115 miles directly east of Columbus, Ohio. The bearing from Columbus to Richmond, Virginia is $S 60^\circ E$. The bearing from Philadelphia to Richmond is $S 34^\circ W$. Find the distance from Columbus to Richmond to the nearest mile.

- A. 314 miles
- B. 321 miles
- C. 332 miles
- D. 345 miles

27. Find the area of the triangle below.



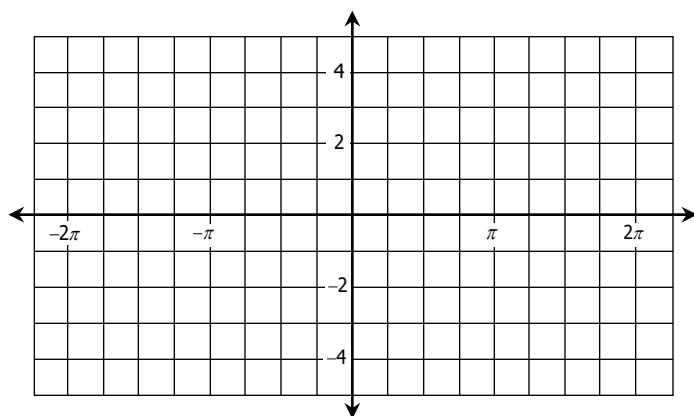
- A. 73.1 km^2
- B. 78.9 km^2
- C. 82.4 km^2
- D. 85.2 km^2

28. A triangular parking lot has sides measuring 90 meters, 54 meters, and 128 meters. Use Heron's Formula to find the area of the parking lot.

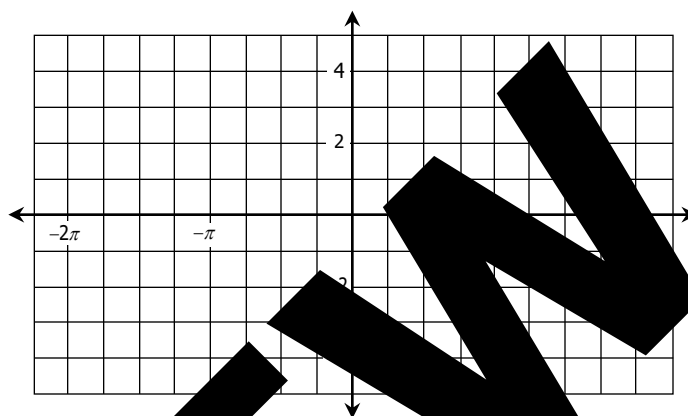
- A. $1,805.2 \text{ m}^2$
- B. $1,992.7 \text{ m}^2$
- C. $2,025.8 \text{ m}^2$
- D. $2,308.6 \text{ m}^2$

Identify the amplitude and period of each function, then graph.

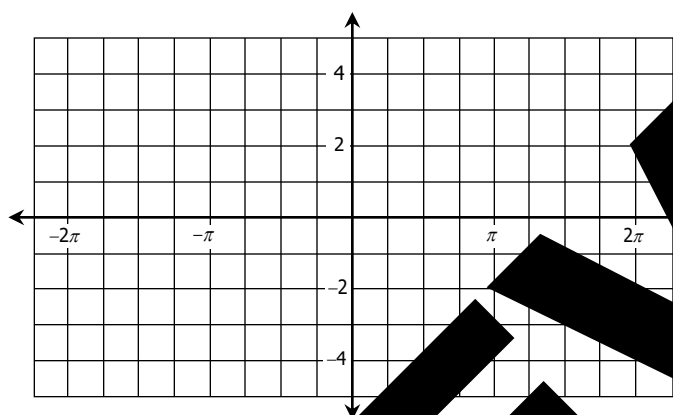
29. $f(x) = 2 \cdot \cos \frac{4}{3}x$ Amp: _____ Per: _____



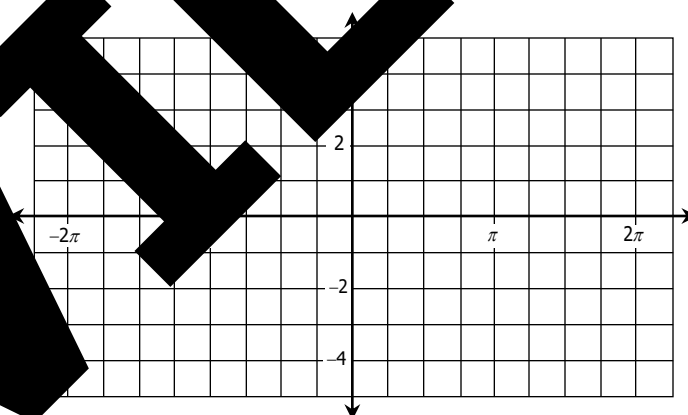
30. $f(x) = \frac{1}{2} \cdot \csc 2x$ Amp: _____ Per: _____



31. $f(x) = -3 \cdot \cot \frac{1}{2}x$ Amp: _____ Per: _____

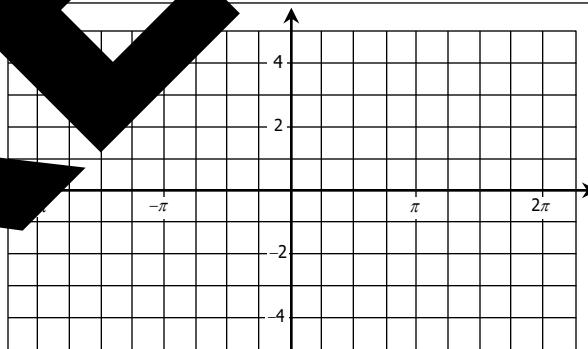


32. $f(x) = 8 \cdot \sec 2x$ Amp: _____ Per: _____



Identify the amplitude, period, phase shifts, and midline of each function. Then graph.

33. $f(x) = \frac{3}{2} \cdot \sec \left(\frac{4}{3}(x + \pi) \right) + 1$



Amplitude:

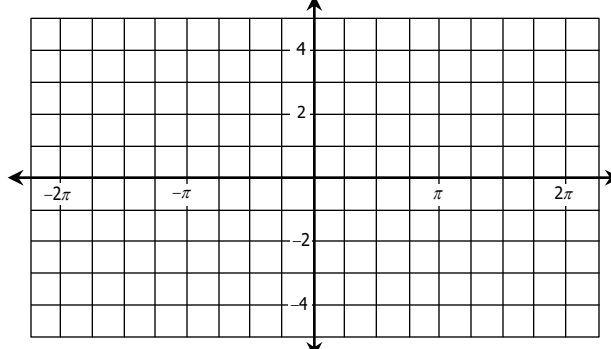
Period:

Phase Shift:

Vertical Shift:

Midline:

34. $f(x) = \tan \frac{4}{3}(x + \pi) - 2$



Amplitude:

Period:

Phase Shift:

Vertical Shift:

Midline:

For questions 35-42, give the exact value, if it exists.

35. $\arccos\left(-\frac{1}{2}\right)$

A. $-\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

B. $\frac{\pi}{6}$

D. $\frac{5\pi}{6}$

36. $\tan^{-1}\frac{\sqrt{3}}{3}$

A. $\frac{\pi}{6}$

C. $-\frac{\pi}{6}$

B. $\frac{\pi}{3}$

37. $\tan^{-1}\left(\tan\frac{11\pi}{6}\right)$

A. $-\frac{\pi}{6}$

C. $\frac{\pi}{4}$

B. $-\frac{\pi}{3}$

D. Not possible

38. $\cos^{-1}(\tan^{-1}(1))$

A. 0

C. $\frac{\pi}{2}$

B. π

D. Not possible

39. $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$

A. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

D. Not possible

40. $\tan[\sin^{-1}(-1)]$

A. 0

C. -1

B. 1

D. Not possible

41. $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

A. $-\frac{1}{2}$

C. $-\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

D. Not possible

42. $\sin\left[\arccos\left(-\frac{\sqrt{7}}{5}\right)\right]$

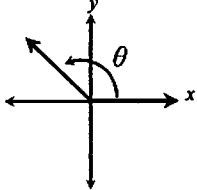
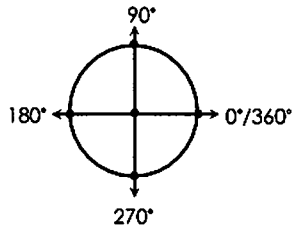
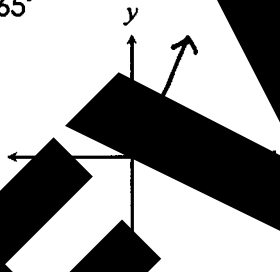
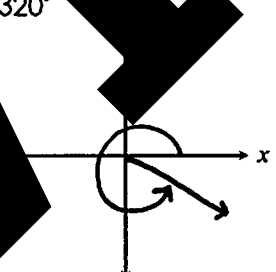
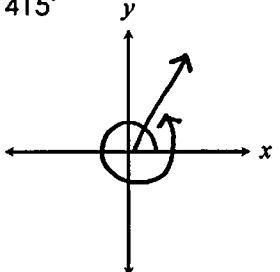
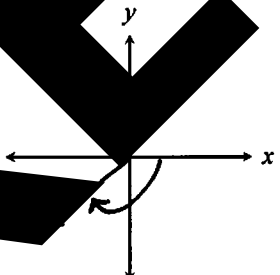
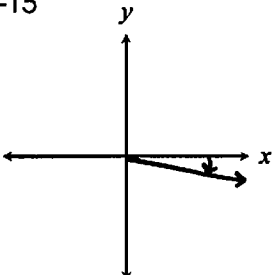
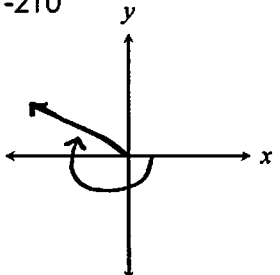
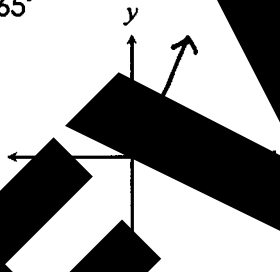
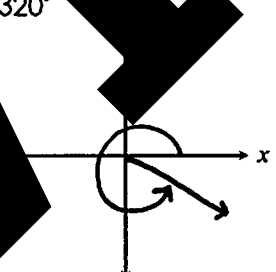
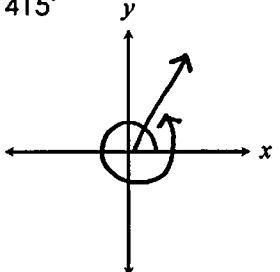
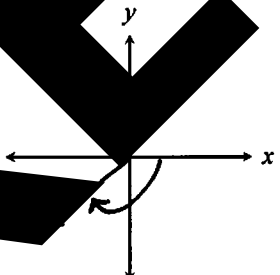
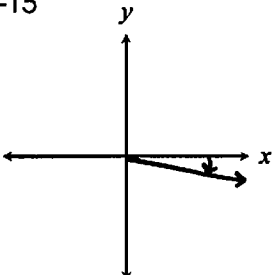
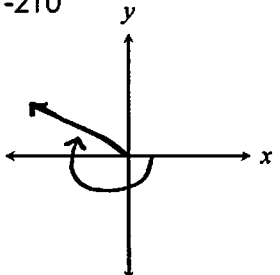
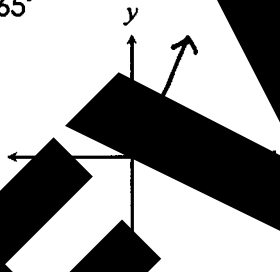
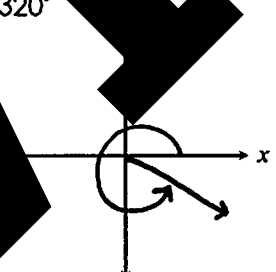
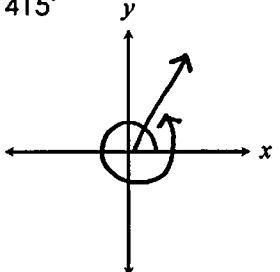
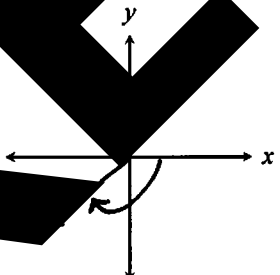
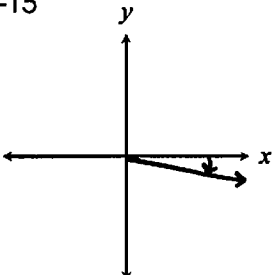
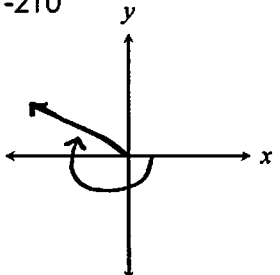
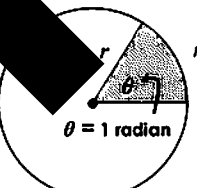
A. $\frac{4\sqrt{2}}{5}$

C. $\frac{3\sqrt{2}}{5}$

B. $-\frac{4\sqrt{2}}{5}$

D. $-\frac{3\sqrt{2}}{5}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples								
<i>Angles in</i> STANDARD FORM 	- An angle on the coordinate plane is in standard form if the vertex is on the origin and one ray lies on the positive x-axis. - The ray on the x-axis is called the <u>initial side</u>. - The other ray is called the <u>terminal side</u>. - Counterclockwise rotations result in <u>positive</u> angle measures. - Clockwise rotations result in <u>negative</u> angle measures.								
DEGREES 	- The most common unit of angle measure for angles is the <u>degree</u> (°). - A degree is equivalent to <u>1/360</u> of a full rotation. Sketch an angle in standard form for the given measure. <table><tr><td>1. 65° </td><td>2. 320° </td><td>3. 415° </td></tr><tr><td>4. -65° </td><td>5. -15° </td><td>6. -210° </td></tr></table>			1. 65° 	2. 320° 	3. 415° 	4. -65° 	5. -15° 	6. -210° 
1. 65° 	2. 320° 	3. 415° 							
4. -65° 	5. -15° 	6. -210° 							
RADIANS 	A radian is a unit of angle measure based on arc length. One radian is defined as the measure of the angle formed when the radius is equivalent to the length of the intercepted arc. Recall that the circumference of a circle is $2\pi r$, therefore: $360^\circ = \underline{2\pi \text{ radians}}$; $180^\circ = \underline{\pi \text{ radians}}$ <table><tr><th>Converting Degrees → Radians</th><th>Converting Radians → Degrees</th></tr><tr><td>$\text{Radians} = \text{Degrees} \cdot \left(\frac{\pi \text{ radians}}{180} \right)$</td><td>$\text{Degrees} = \text{Radians} \cdot \left(\frac{180}{\pi \text{ radians}} \right)$</td></tr></table>			Converting Degrees → Radians	Converting Radians → Degrees	$\text{Radians} = \text{Degrees} \cdot \left(\frac{\pi \text{ radians}}{180} \right)$	$\text{Degrees} = \text{Radians} \cdot \left(\frac{180}{\pi \text{ radians}} \right)$		
Converting Degrees → Radians	Converting Radians → Degrees								
$\text{Radians} = \text{Degrees} \cdot \left(\frac{\pi \text{ radians}}{180} \right)$	$\text{Degrees} = \text{Radians} \cdot \left(\frac{180}{\pi \text{ radians}} \right)$								

Converting DEGREES TORADIANS	Convert each angle measure to radians.		
	7. 60° $60 \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{3}}$	8. 160° $160 \cdot \frac{\pi}{180} = \boxed{\frac{8\pi}{9}}$	9. 330° $330 \cdot \frac{\pi}{180} = \boxed{\frac{11\pi}{6}}$
	10. -45° $-45 \cdot \frac{\pi}{180} = \boxed{-\frac{\pi}{4}}$	11. -230° $-230 \cdot \frac{\pi}{180} = \boxed{-\frac{23\pi}{18}}$	12. -75° $-75 \cdot \frac{\pi}{180} = \boxed{-\frac{5\pi}{12}}$
Converting RADIANSTODEGREES	Convert each angle measure to degrees		
	13. $\frac{5\pi}{18}$ $\frac{5\pi}{18} \cdot \frac{180}{\pi} = \boxed{50^\circ}$	14. $\frac{11\pi}{9}$ $\frac{11\pi}{9} \cdot \frac{180}{\pi} = \boxed{220^\circ}$	15. $\frac{7\pi}{3}$ $\frac{7\pi}{3} \cdot \frac{180}{\pi} = \boxed{420^\circ}$
	16. $-\frac{\pi}{6}$ $-\frac{\pi}{6} \cdot \frac{180}{\pi} = \boxed{-30^\circ}$	17. $-\frac{19\pi}{2}$ $-\frac{19\pi}{2} \cdot \frac{180}{\pi} = \boxed{-1530^\circ}$	18. $-\frac{3\pi}{4}$ $-\frac{3\pi}{4} \cdot \frac{180}{\pi} = \boxed{-135^\circ}$
COTERMINAL ANGLES	Angles in standard position with the same terminal side are coterminal angles. Determine if the two angles are coterminal.		
	19. 180° and -55° $180 - 55 = 125 \neq 0$ No!	20. 30° and -170° $30 - 360 = -330 \neq -170$ No!	
	21. $\frac{\pi}{4}$ and $\frac{9\pi}{4}$ $\frac{\pi}{4} - 2\pi = -\frac{7\pi}{4} \neq \frac{9\pi}{4}$ No!	22. $\frac{3\pi}{2}$ and $-\frac{5\pi}{2}$ $\frac{3\pi}{2} - 4\pi = -\frac{5\pi}{2}$ Yes!	
	Find coterminal angles for each given angle, one positive and one negative.		
	23. 150° $150^\circ + 360^\circ = \boxed{510^\circ}$ $150^\circ - 360^\circ = \boxed{-210^\circ}$	24. 265° $265^\circ + 360^\circ = \boxed{625^\circ}$ $265^\circ - 360^\circ = \boxed{-95^\circ}$	25. -25° $-25^\circ + 360^\circ = \boxed{335^\circ}$ $-25^\circ - 360^\circ = \boxed{-385^\circ}$
	26. $\frac{\pi}{12}$ $\frac{\pi}{12} + 2\pi = \boxed{\frac{25\pi}{12}}$ $\frac{\pi}{12} - 2\pi = \boxed{-\frac{23\pi}{12}}$	27. $\frac{7\pi}{6}$ $\frac{7\pi}{6} + 2\pi = \boxed{\frac{19\pi}{6}}$ $\frac{7\pi}{6} - 2\pi = \boxed{-\frac{5\pi}{6}}$	28. $-\frac{11\pi}{18}$ $-\frac{11\pi}{18} + 2\pi = \boxed{\frac{25\pi}{18}}$ $-\frac{11\pi}{18} - 2\pi = \boxed{-\frac{47\pi}{18}}$

Name:	Date:
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Main Ideas/Questions	Notes/Examples						
DEGREE-MINUTE-SECOND FORM (DMS)	Some angles have decimal degrees (for example, 26.85°). Angles with decimal degrees can be expressed using degree-minute-second (DMS) form, in which the degrees are subdivided into minutes and seconds. one degree = <u>60</u> minutes; one minute = <u>60</u> seconds *one degree = <u>3600</u> seconds Symbols for degree, minutes, and seconds: <table><tr><th>DEGREE</th><th>MINUTE</th><th>SECOND</th></tr><tr><td>$^\circ$</td><td>$'$</td><td>$''$</td></tr></table>	DEGREE	MINUTE	SECOND	$^\circ$	$'$	$''$
DEGREE	MINUTE	SECOND					
$^\circ$	$'$	$''$					
Converting DECIMAL DEGREES TO DMS	Follow the example below to convert 31.875° to degree-minute-second form. $31.875^\circ = 31^\circ + .875^\circ$ $.875^\circ \cdot 60 = 52.5$ $.5^\circ \cdot 60 = 30$ Therefore, $31.875^\circ = 31^\circ 52' 30''$ Convert the angle measure in DMS form. <table><tr><td> 1. 48.815° $.815^\circ \cdot 60 = 48.9$ $.9^\circ \cdot 60 = 54$ $48^\circ 54' 54''$ </td><td> 2. 150.2° $.2(60) = 12$ $150^\circ 12'$ </td></tr><tr><td> 3. 89.915° $.915(60) = 54.9$ $.9(60) = 54$ $89^\circ 54' 54''$ </td><td> 4. 340.375° $.375(60) = 22.5$ $.5(60) = 30$ $340^\circ 22' 30''$ </td></tr><tr><td> 5. 48.526° $.526(60) = 31.56$ $.56(60) = 33.6$ $48^\circ 31' 34''$ </td><td> 6. 273.771° $.771(60) = 46.26$ $.26(60) = 15.6$ $273^\circ 46' 16''$ </td></tr></table>	1. 48.815° $.815^\circ \cdot 60 = 48.9$ $.9^\circ \cdot 60 = 54$ $48^\circ 54' 54''$	2. 150.2° $.2(60) = 12$ $150^\circ 12'$	3. 89.915° $.915(60) = 54.9$ $.9(60) = 54$ $89^\circ 54' 54''$	4. 340.375° $.375(60) = 22.5$ $.5(60) = 30$ $340^\circ 22' 30''$	5. 48.526° $.526(60) = 31.56$ $.56(60) = 33.6$ $48^\circ 31' 34''$	6. 273.771° $.771(60) = 46.26$ $.26(60) = 15.6$ $273^\circ 46' 16''$
1. 48.815° $.815^\circ \cdot 60 = 48.9$ $.9^\circ \cdot 60 = 54$ $48^\circ 54' 54''$	2. 150.2° $.2(60) = 12$ $150^\circ 12'$						
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5. 48.526° $.526(60) = 31.56$ $.56(60) = 33.6$ $48^\circ 31' 34''$	6. 273.771° $.771(60) = 46.26$ $.26(60) = 15.6$ $273^\circ 46' 16''$						

$$7. 105.394^{\circ}$$

$$.394(60) = 23.64$$

$$.64(60) = 38.4$$

$$105^{\circ} 23' 38''$$

$$8. 318.812^{\circ}$$

$$.812(60) = 48.72$$

$$.72(60) = 43.2$$

$$318^{\circ} 48' 43''$$

Converting DMS TO DECIMAL DEGREES

Follow the example below to convert $73^{\circ} 9' 50''$ to decimal degrees.

$$73^{\circ} 9' 50'' \rightarrow 9 / 60 = .15$$

$$50 / 60 = .8\overline{3}$$

$$\text{Therefore, } 73^{\circ} 9' 50'' = 73 + 0.15 + .8\overline{3} = 73.163\overline{3}^{\circ}$$

Write each angle measure in decimal degrees.

$$9. 37^{\circ} 36'$$

$$36 / 60 = 0.6$$

$$37 + 0.6 = 37.6^{\circ}$$

$$10. 315^{\circ} 8'$$

$$8 / 60 = 0.133\overline{3}$$

$$315 + 0.133\overline{3} = 315.133\overline{3}^{\circ}$$

$$11. 84^{\circ} 32' 18''$$

$$32 / 60 = 0.533\overline{3}$$

$$18 / 3600 = 0.005$$

$$84 + 0.533\overline{3} + 0.005 = 84.538\overline{3}^{\circ}$$

$$12. 121^{\circ} 4' 45''$$

$$4 / 60 = 0.066\overline{7}$$

$$45 / 3600 = 0.0125$$

$$121 + 0.066\overline{7} + 0.0125 = 121.0792^{\circ}$$

$$13. 289^{\circ} 28' 20''$$

$$28 / 60 = 0.466\overline{7}$$

$$20 / 3600 = 0.0056$$

$$289 + 0.466\overline{7} + 0.0056 = 289.4723^{\circ}$$

$$14. 8^{\circ} 34' 16''$$

$$34 / 60 = 0.566\overline{7}$$

$$16 / 3600 = 0.0044$$

$$8 + 0.566\overline{7} + 0.0044 = 8.5711^{\circ}$$

$$15. 50^{\circ} 14' 56''$$

$$14 / 60 = 0.233\overline{3}$$

$$56 / 3600 = 0.0156$$

$$50 + 0.233\overline{3} + 0.0156 = 50.2489^{\circ}$$

$$16. 253^{\circ} 46' 5''$$

$$46 / 60 = 0.766\overline{7}$$

$$5 / 3600 = 0.0014$$

$$253 + 0.766\overline{7} + 0.0014 = 253.7681^{\circ}$$

Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 1: Angle Measures

**** This is a 2-page document! ******Directions:** Convert each angle measure to radians.1. 140°

$$140 \cdot \frac{\pi}{180} = \boxed{\frac{7\pi}{9}}$$

2. -75°

$$-75 \cdot \frac{\pi}{180} = \boxed{-\frac{5\pi}{12}}$$

3. 190°

$$190 \cdot \frac{\pi}{180} = \boxed{\frac{19\pi}{18}}$$

4. 312°

$$312 \cdot \frac{\pi}{180} = \boxed{\frac{26\pi}{15}}$$

5. -65°

$$-65 \cdot \frac{\pi}{180} = \boxed{-\frac{13\pi}{36}}$$

6. -205°

$$-205 \cdot \frac{\pi}{180} = \boxed{-\frac{41\pi}{36}}$$

Directions: Convert each angle measure to degrees.7. $\frac{3\pi}{5}$

$$\frac{3\pi}{5} \cdot \frac{180}{\pi} = \boxed{108^\circ}$$

8. $-\frac{7\pi}{18}$

$$-\frac{7\pi}{18} \cdot \frac{180}{\pi} = \boxed{-70^\circ}$$

9. $\frac{10\pi}{9}$

$$\frac{10\pi}{9} \cdot \frac{180}{\pi} = \boxed{200^\circ}$$

10. $-\frac{9\pi}{5}$

$$-\frac{9\pi}{5} \cdot \frac{180}{\pi} = \boxed{-324^\circ}$$

11. $\frac{11\pi}{12}$

$$\frac{11\pi}{12} \cdot \frac{180}{\pi} = \boxed{165^\circ}$$

12. $\frac{15\pi}{4}$

$$\frac{15\pi}{4} \cdot \frac{180}{\pi} = \boxed{675^\circ}$$

Directions: Find two coterminal angles for each given angle, one positive and one negative.13. 108°

$$108 + 360 = \boxed{468^\circ}$$

$$108 - 360 = \boxed{-252^\circ}$$

14. 319°

$$319 + 360 = \boxed{679^\circ}$$

$$319 - 360 = \boxed{-41^\circ}$$

15. -64°

$$-64 + 360 = \boxed{296^\circ}$$

$$-64 - 360 = \boxed{-424^\circ}$$

16. $\frac{3\pi}{5}$

$$\frac{3\pi}{5} + 2\pi = \boxed{\frac{13\pi}{5}}$$

$$\frac{3\pi}{5} - 2\pi = \boxed{-\frac{7\pi}{5}}$$

17. $\frac{11\pi}{7}$

$$\frac{11\pi}{7} + 2\pi = \boxed{\frac{25\pi}{7}}$$

$$\frac{11\pi}{7} - 2\pi = \boxed{-\frac{3\pi}{7}}$$

18. $\frac{19\pi}{6}$

$$\frac{19\pi}{6} - 2\pi = \boxed{\frac{7\pi}{6}}$$

$$\frac{19\pi}{6} - 4\pi = \boxed{-\frac{5\pi}{6}}$$

Directions: Write each angle measure in Degree-Minute-Second (DMS) form.

19. 36.875°

$.875(60) = 52.5$

$.5(60) = 30$

$36^\circ 52' 30''$

20. 117.590°

$.590(60) = 35.4$

$.4(60) = 24$

$117^\circ 35' 24''$

21. -238.145°

$.145(60) = 8.7$

$.7(60) = 42$

$-238^\circ 8' 42''$

22. 64.901°

$.901(60) = 54.06$

$.06(60) = 3.6$

$64^\circ 54' 4''$

23. -323.588°

$.588(60) = 35.28$

$.28(60) = 16.8$

$-323^\circ 35' 17''$

24. 136°

$.36(60) = 21.6$

$.16(60) = 9.6$

$136^\circ 21' 10''$

Directions: Write each angle measure in decimal degree form.

25. $54^\circ 26' 18''$

$26/60 = .4333$

$18/3600 = .005$

54.4383°

$184^\circ 37' 20''$

$37/60 = .6167$

$20/3600 = .0056$

184.6223°

27. -98°

$12/60 = .2$

$10/60 = .1667$

-98.2133°

28. $289^\circ 5' 30''$

$5/60 = .0833$

$30/3600 = .0083$

289.0916°

29. $-303^\circ 20' 14''$

$20/60 = .3333$

$14/3600 = .0039$

-303.3372°

30. $100^\circ 45' 6''$

$45/60 = .75$

$6/3600 = .0017$

100.7517°

Name:

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Main Ideas/Questions

Notes/Examples

ARC LENGTH

If θ is a central angle in a circle with radius r , then the length of the intercepted arc, s , is given by

$$S = r\theta$$

(where θ is measured in radians)

Examples

Find the length of the intercepted arc given the angle measure and radius. Give your answer in terms of π and round to the nearest tenth.

1. $\frac{5\pi}{6}; r = 3 \text{ ft}$

$$S = 3\left(\frac{5\pi}{6}\right) = \frac{5\pi}{2} \text{ ft}$$

$$= 7.85 \text{ ft}$$

2. $\frac{\pi}{8}; r = 4.5 \text{ in}$

$$S = 4.5\left(\frac{\pi}{8}\right) = \frac{29\pi}{8} \text{ in}$$

$$= 11.4 \text{ in}$$

3. $\frac{3\pi}{2}; r = 0.5 \text{ yd}$

$$S = 0.5\left(\frac{3\pi}{2}\right) = \frac{3\pi}{4} \text{ yd}$$

$$= 2.4 \text{ yd}$$

4. $2\pi; r = 6.2 \text{ yd}$

$$S = 6.2(2\pi) = \frac{62\pi}{5} \text{ yd}$$

$$= 39 \text{ yd}$$

Convert to radians

5. $120^\circ; r = 8.4 \text{ m}$

$$S = 8.4\left(\frac{2\pi}{3}\right) = \frac{7\pi}{10} \text{ m}$$

$$= 2.2 \text{ m}$$

6. $330^\circ; r = 10.5 \text{ mm}$

$$\hookrightarrow \frac{11\pi}{6}$$

$$S = 10.5\left(\frac{11\pi}{6}\right) = \frac{77\pi}{4} \text{ mm}$$

$$= 60.5 \text{ mm}$$

7. $120^\circ; r = 1.8 \text{ ft}$

$$\hookrightarrow \frac{2\pi}{3}$$

$$S = 1.8\left(\frac{2\pi}{3}\right) = \frac{6\pi}{5} \text{ ft}$$

$$= 3.8 \text{ ft}$$

8. $135^\circ; r = 25 \text{ km}$

$$\hookrightarrow \frac{3\pi}{4}$$

$$S = 25\left(\frac{3\pi}{4}\right) = \frac{75\pi}{4} \text{ km}$$

$$= 58.9 \text{ km}$$

9. The central angle θ in a circle of radius 6 meters has an intercepted arc length of 10 meters. Find the measure of θ in radians and in degrees.

$$10 = \theta 6$$

$$\theta = \frac{5}{3} \text{ radians}$$

$$\frac{5}{3} \cdot \frac{180}{\pi} = 95.5^\circ$$

10. A merry go round rotates 2808° per ride. How far would a rider seated 8 feet from the center of the merry go round travel during the ride?

$$2808^\circ \rightarrow \frac{78\pi}{5}$$

$$S = 8 \left(\frac{78\pi}{5} \right) = 392.1 \text{ ft}$$

11. Cincinnati, Ohio is directly north of Atlanta, Georgia. Cincinnati has a latitude of 39.1° N and Atlanta has a latitude of 33.7° N . The earth has a radius of 3,960 miles, how far apart are these cities?

$$5.4^\circ \rightarrow \frac{3\pi}{100}$$

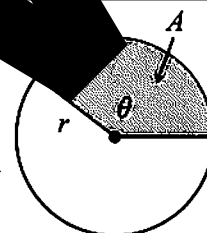
$$S = 3960 \left(\frac{3\pi}{100} \right) = 362.2 \text{ miles}$$

AREA OF SECTORS

The area A of a sector of a circle with radius r and central angle θ is given by:

$$A = \frac{1}{2} \theta r^2$$

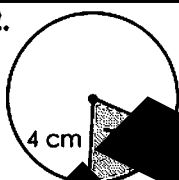
(where θ is measured in radians)



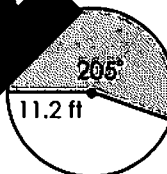
Examples

Find the area of each sector.

12.



$$\begin{aligned} A &= \frac{1}{2} \left(\frac{37\pi}{180} \right) (4)^2 \\ &= \frac{37\pi}{160} (16) \\ &= 3.68 \text{ cm}^2 \end{aligned}$$

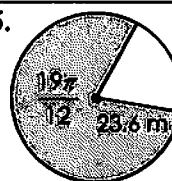


$$\begin{aligned} A &= \frac{1}{2} \left(\frac{41\pi}{36} \right) (11.2)^2 \\ &= \frac{41\pi}{72} (125.44) \\ &= 224.4 \text{ ft}^2 \end{aligned}$$



$$\begin{aligned} A &= \frac{1}{2} \left(\frac{2\pi}{3} \right) (7)^2 \\ &= \frac{\pi}{3} (49) \\ &= 51.3 \text{ km}^2 \end{aligned}$$

15.



$$\begin{aligned} A &= \frac{1}{2} \left(\frac{19\pi}{2} \right) (23.6)^2 \\ &= \frac{19\pi}{4} (556.96) \\ &= 1385.2 \text{ m}^2 \end{aligned}$$

16. The area of a sector of a circle with a central angle of 240° is 134 ft^2 . Find the radius of the circle.

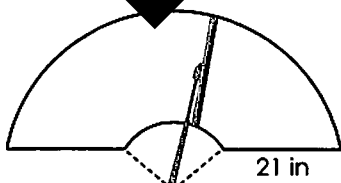
$$134 = \frac{1}{2} \left(\frac{4\pi}{3} \right) r^2$$

$$134 = 2.1 r^2$$

$$64 = r^2$$

$$r = 8 \text{ ft}$$

$$\hookrightarrow \frac{4\pi}{3}$$



17. The windshield wiper arm to the left is 32 inches long. If the wiper sweeps through an angle of 125° , find the area swept by the blade.

$$125^\circ = \frac{25\pi}{36}$$

$$\begin{aligned} A_1 &= \frac{1}{2} \left(\frac{25\pi}{36} \right) (32)^2 \\ &= \frac{25\pi}{72} (1024) \\ &= 1117 \end{aligned}$$

$$\begin{aligned} A_2 &= \frac{1}{2} \left(\frac{25\pi}{36} \right) (11)^2 \\ &= \frac{25\pi}{72} (121) \\ &= 132 \end{aligned}$$

$$A = 1117 - 132 = 985 \text{ in}^2$$

Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 2: Arc Lengths & Area of Sectors

** This is a 2-page document! **

Directions: Find the length of each intercepted arc given the angle measure and radius. Give your answer in terms of π and rounded to the nearest tenth.

1. $\frac{3\pi}{4}; r = 10 \text{ mm}$

$$S = 10 \left(\frac{3\pi}{4} \right) = \frac{15\pi}{2} \text{ mm}$$

$$= 23.6 \text{ mm}$$

2. $\frac{8\pi}{7}; r = 17.5 \text{ yd}$

$$S = 17.5 \left(\frac{8\pi}{7} \right) = 20\pi \text{ yd}$$

$$= 62.8 \text{ yd}$$

3. $\frac{\pi}{6}; r = 2 \text{ mi}$

$$S = 2 \left(\frac{\pi}{6} \right) = \frac{\pi}{3} \text{ mi}$$

$$= 1.0 \text{ mi}$$

4. $\frac{5\pi}{8}; r = 18 \text{ ft}$

$$S = 18 \left(\frac{5\pi}{8} \right) = \frac{45\pi}{4} \text{ ft}$$

$$= 35.3 \text{ ft}$$

5. $320^\circ; r = 0.9 \text{ cm}$

$$\hookrightarrow \frac{16\pi}{9}$$

$$S = 0.9 \left(\frac{16\pi}{9} \right) = \frac{8\pi}{5} \text{ cm}$$

$$= 5.0 \text{ cm}$$

6. $100^\circ; r = 4 \text{ yd}$

$$\hookrightarrow \frac{5\pi}{9}$$

$$S = 4 \left(\frac{5\pi}{9} \right) = \frac{20\pi}{9} \text{ yd}$$

$$= 7.9 \text{ yd}$$

7. $240^\circ; r = 6.6 \text{ km}$

$$\hookrightarrow \frac{4\pi}{3}$$

$$S = 6.6 \left(\frac{4\pi}{3} \right) = \frac{44\pi}{5} \text{ km}$$

$$= 27.6 \text{ km}$$

8. $45^\circ; r = 2.2 \text{ m}$

$$\hookrightarrow \frac{\pi}{4}$$

$$S = 2.2 \left(\frac{\pi}{4} \right) = \frac{11\pi}{20} \text{ m}$$

$$= 1.7 \text{ m}$$

9. An intercepted arc has a length of 19 yards. If the radius is 4 yards, find the measure of the angle in radians and degrees.

$$19 = 4\theta$$

$$\theta = \frac{19}{4} \text{ radians}$$

$$\frac{19}{4} \cdot \frac{180}{\pi} = 272.2^\circ$$

10. The length of an hour hand on a clock is 5.5 inches from the center. How far will the hour hand travel over three days?

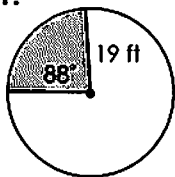
$$\hookrightarrow 360 \cdot 6 = 2160^\circ \rightarrow 12\pi$$

$$S = 5.5(12\pi) = 66\pi \text{ in}$$

$$= 207.3 \text{ in}$$

Directions: Find the area of each sector.

11.

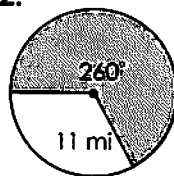


$$A = \frac{1}{2} \left(\frac{22\pi}{45} \right) (19)^2$$

$$= \frac{11\pi}{45} (361)$$

$$= \boxed{277.2 \text{ ft}^2}$$

12.

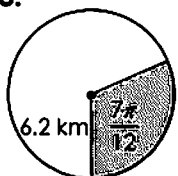


$$A = \frac{1}{2} \left(\frac{13\pi}{9} \right) (11)^2$$

$$= \frac{13\pi}{18} (121)$$

$$= \boxed{277.2 \text{ mi}^2}$$

13.

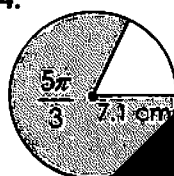


$$A = \frac{1}{2} \left(\frac{7\pi}{12} \right) (6.2)^2$$

$$= \frac{7\pi}{24} (38.44)$$

$$= \boxed{35.2 \text{ km}^2}$$

14.

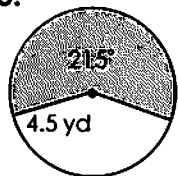


$$A = \frac{1}{2} \left(\frac{5\pi}{3} \right) (7.1)^2$$

$$= \frac{5\pi}{6} (50.41)$$

$$= \boxed{132 \text{ cm}^2}$$

15.



$$A = \frac{1}{2} \left(\frac{43\pi}{36} \right) (4.5)^2$$

$$= \frac{43\pi}{72} (20.25)$$

$$= \boxed{38 \text{ yd}^2}$$



$$A = \frac{1}{2} \left(\frac{13\pi}{18} \right) (21.4)^2$$

$$= \frac{13\pi}{36} (457.96)$$

$$= \boxed{519.5 \text{ m}^2}$$

17. The sector of a circle has a central angle of $\frac{2\pi}{5}$ radians and an area of 70.7 square meters. Find the radius of the circle.

$$70.7 = \frac{1}{2} \left(\frac{2\pi}{5} \right) r^2$$

$$70.7 = \frac{\pi}{5} r^2$$

$$= 56.2$$

$$r = \boxed{7.5 \text{ m}}$$

18. A circle with a radius of 15 yd contains a sector with an area of 609 yd². Find the measure of the central angle for the sector in both radians and degrees.

$$609 = \frac{1}{2} \theta \cdot 15^2$$

$$609 = \frac{1}{2} \theta \cdot 225$$

$$\theta = \frac{406}{75} \text{ radians}$$

$$\frac{129.2\pi}{75} \cdot \frac{180}{\pi} = \boxed{310^\circ}$$

19. A clock is equally divided into 12 sections. If the clock reads 8:00 and has a diameter of 12.5 inches, find the area of the smaller sector formed by the minute and hour hands.

Section = $30^\circ \rightarrow \frac{\pi}{6}$

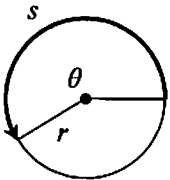
Sector = $4 \cdot \frac{\pi}{6} = \frac{2\pi}{3}$

$$A = \frac{1}{2} \left(\frac{2\pi}{3} \right) (6.25)^2$$

$$= \frac{\pi}{3} (39.0625)$$

$$= \boxed{40.9 \text{ in}^2}$$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
CIRCULAR MOTION 	Arc length can be used to analyze circular motion. Suppose an object moves along a circular path with radius r.	
	The rate at which the object moves along the path is called its linear speed, v.	The rate at which the angle is changing along the path is called its angular speed, ω.
	Linear Speed Formula: $v = \frac{s}{t} \left(\frac{\text{Arc length}}{\text{time}} \right)$	Angular Speed Formula: $\omega = \frac{\theta}{t} \left(\frac{\text{Central angle}}{\text{time}} \right)$
Examples Recall: Each rotation (or revolution) has a circumference of $2\pi r$ and a central angle of 2π radians.	Use for questions 1-2: A bicycle tire with a diameter of 14 inches rotates at a rate of 125 revolutions per minute.	
	1. Find the linear speed of the tire in inches per minute. $S = 2\pi(14)(125) = 3500\pi \text{ in}$ $v = \frac{3500\pi \text{ in}}{1 \text{ min}} \approx 10996 \text{ in/min}$	2. Find the angular speed of the tire in radians per minute. $\theta = 125(2\pi) = 250\pi$ $\omega = \frac{250\pi}{1 \text{ min}} \approx 785.4 \text{ radians/min}$
	Use for questions 3-4: A CD with a diameter of 120 millimeters rotates a rate of 45 revolutions per minute.	
	3. Find the linear speed of the CD in millimeters per minute. $S = 2\pi(60)(45) = 5400\pi \text{ mm}$ $v = \frac{5400\pi \text{ mm}}{1 \text{ min}} \approx 16,964.6 \text{ mm/min}$	4. Find the angular speed of the CD in radians per minute. $\theta = 45(2\pi) = 90\pi$ $\omega = \frac{90\pi}{1 \text{ min}} \approx 282.7 \text{ radians/min}$
	5. A 16-inch diameter tire on a car is making 500 revolutions per minute. Find the linear speed of the tire in miles per hour.	
	$S = 2\pi(8)(500) = 8000\pi \text{ in}; v = \frac{8000\pi \text{ in}}{1 \text{ min}}$ $\frac{8000\pi \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 23.8 \text{ mi/hr}$	

6. A circular saw blade with a diameter of 9 inches rotates at 2800 revolutions per minute. Find the angular speed of the blade in radians per second.

$$\theta = 2\pi(2800) = 5600\pi \text{ radians}; w = \frac{5600\pi \text{ radians}}{1 \text{ min}}$$

$$\frac{5600\pi \text{ rad}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx \boxed{293.2 \text{ rad/sec}}$$

7. A windmill has blades that are 14 feet long. If the windmill is rotating at 5 revolutions per second, find the linear speed of the tips of the blades in miles per hour.

$$S = 2\pi(14)(5) = 140\pi \text{ ft/sec} = \frac{140\pi \text{ ft}}{1 \text{ sec}}$$

$$\frac{140\pi \text{ ft}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx \boxed{299.9 \text{ mi/hr}}$$

Using
ANGULAR SPEED
to find
LINEAR SPEED

The linear speed, v , can be found as follows:

$$\frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = rw$$

Therefore, you can use the angular speed, w , to find the linear speed, v .

Example: A windmill with 25-inch blades rotates at 40 rpm. Find the linear speed of the tips of the blades in feet per second.

$$w = 2\pi(40) = 80\pi \text{ radians/min}$$

$$v = 80\pi(25) = 2000\pi \text{ in/min}$$

$$\frac{2000\pi \text{ in}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \approx \boxed{8.7 \text{ ft/s}}$$

9. Ryan is riding a bicycle whose wheels are 28 inches in diameter. If the wheels rotate at 130 rpm, find the linear speed in miles per hour in which he is traveling.

$$w = 2\pi(130) = 260\pi \text{ radians/min}$$

$$v = 260\pi(14) = 3640\pi \text{ in/min}$$

$$\frac{3640\pi \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx \boxed{10.8 \text{ mi/hr}}$$

Name: _____

Unit 5: Trigonometric Functions

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Homework 3: Linear & Angular Speeds

**** This is a 2-page document! ****

Use for questions 1 and 2: An SUV tire with a radius of 12.5 inches rotates at a rate of 545 revolutions per minute.

1. Find the linear speed of the tire in inches per minute.

$$S = 2\pi(12.5)(545) \\ = 13625\pi \text{ in}$$

$$V = \frac{13625\pi \text{ in}}{1 \text{ min}} \approx \boxed{42804.2 \text{ in/min}}$$

2. Find the angular speed of the tire in radians per minute.

$$\theta = 545(2\pi) = 1090\pi \text{ rad}$$

$$W = \frac{1090\pi \text{ rad}}{1 \text{ min}} \approx \boxed{3420 \text{ rad/min}}$$

Use for questions 3 and 4: The fan on a pool heater has a diameter of 53.5 centimeters and rotates at a rate of 212 revolutions per minute.

3. Find the linear speed of the fan in centimeters per minute.

$$S = 2\pi(53.5)(212) \\ = 22684\pi \text{ cm}$$

$$V = \frac{22684\pi \text{ cm}}{1 \text{ min}} \approx \boxed{71363 \text{ cm/min}}$$

4. Find the angular speed of the fan in radians per minute.

$$\theta = 212(2\pi) = 424\pi \text{ rad}$$

$$W = \frac{424\pi \text{ rad}}{1 \text{ min}} \approx \boxed{1332 \text{ rad/min}}$$

5. A skateboard wheel with a 76 millimeter diameter makes 430 revolutions per minute. Find the linear speed of the wheel in meters per second.

$$S = 2\pi(76)(430) = 24080\pi \text{ mm}; \quad V = \frac{24080\pi \text{ mm}}{1 \text{ min}}$$

$$\frac{24080\pi \text{ mm}}{1 \text{ min}} \cdot \frac{1 \text{ m}}{1000 \text{ mm}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \approx \boxed{1.3 \text{ m/sec}}$$

A bicycle tire with a 12-inch radius is making 175 revolutions per minute. Find the linear speed in miles per hour.

$$S = 2\pi(12)(175) = 4200\pi \text{ in}; \quad V = \frac{4200\pi \text{ in}}{1 \text{ min}}$$

$$\frac{4200\pi \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx \boxed{12.5 \text{ mi/hr}}$$

7. A record with a diameter of 12 inches rotates at 33 revolutions per minute. Find the angular speed of the record in radians per second.

$$\theta = 2\pi(33) = 66\pi \text{ radians}; \quad \omega = \frac{66\pi \text{ rad}}{1 \text{ min}}$$

$$\frac{66\pi \text{ rad}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx \boxed{3.5 \text{ rad/sec}}$$

8. An oscillating fan has blades that measure 8 inches long. If the fan rotates at a speed of 9 revolutions per second, find the linear speed of the ends of the blades in miles per hour.

$$S = 2\pi(8)(9) = 144\pi \text{ in}; \quad V = \frac{144\pi \text{ in}}{1 \text{ sec}}$$

$$\frac{144\pi \text{ in}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx \boxed{25.7 \text{ mi/hr}}$$

9. The wheels on Darlene's car have an 11-inch radius. If the wheels are rotating at a rate of 378 rpm, find the linear speed in miles per hour at which she is driving.

$$\omega = 2\pi(378) = 756\pi \text{ rad/min}$$

$$V = 756\pi(11) = 8316\pi \text{ in/min}$$

$$\frac{8316\pi \text{ in}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx \boxed{24.7 \text{ mi/hr}}$$

10. A DVD with a radius of 6 centimeters rotates at a speed of 243 revolutions per minute. Find the linear speed in centimeters per second.

$$\omega = 2\pi(243) = 486\pi \text{ rad/min}$$

$$V = 486\pi(6) = 2916\pi \text{ cm/min}$$

$$\frac{2916\pi \text{ cm}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx \boxed{152.7 \text{ cm/sec}}$$

11. A lawnmower blade has a diameter of 30 inches. If the blade rotates at a speed of 126 revolutions per minute, find the linear speed of the tips of the blades in feet per second.

$$\omega = 2\pi(126) = 252\pi \text{ rad/min}$$

$$V = 252\pi(15) = 3780\pi \text{ in/min}$$

$$\frac{3780\pi \text{ in}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \approx \boxed{16.5 \text{ ft/sec}}$$

Name: _____

Pre-Calculus

Date: _____ Per: _____

Unit 5: Trigonometric Functions

Quiz 5-1: Angle Measures, Arc Lengths, Area of Sectors, & Circular Motion

Write the following angle measures in radians.

1. 165°

$$165 \cdot \frac{\pi}{180}$$

2. -300°

$$-300 \cdot \frac{\pi}{180}$$

Write the following angle measures in degrees.

3. $-\frac{\pi}{10}$

$$-\frac{\pi}{10} \cdot \frac{180}{\pi}$$

4. $\frac{7\pi}{6}$

$$\frac{7\pi}{6} \cdot \frac{180}{\pi}$$

Give one positive and one negative coterminal angle for the given angle.

5. 135°

$$135 + 360$$

$$135 - 360$$

6. $\frac{\pi}{4}$

$$\frac{\pi}{4}$$

Write the following angle measures in degree-minute-second (DMS) form.

7. 42.25°

$$.25(60) = 15$$

8. -210.615°

$$.615(60) = 36.9$$

$$.9(60) = 54$$

Write the following angle measures in decimal degree form.

9. $164^\circ 39'$

$$39/60 = .65$$

10. $8^\circ 15' 54''$

$$15/60 = .25$$

$$54/3600 = .015$$

Find the length of the intercepted arc given the central angle and radius of the circle. Round your answer to the nearest tenth.

11. $\theta = \frac{4\pi}{5}; r = 9 \text{ cm}$

$$S = 9 \left(\frac{4\pi}{5} \right)$$

$$= 22.6$$

12. $\theta = 345^\circ; r = 2.5 \text{ ft}$

$$\hookrightarrow \frac{23\pi}{12}$$

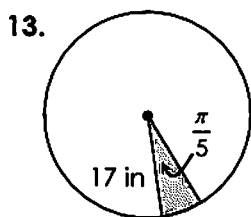
$$S = 2.5 \left(\frac{23\pi}{12} \right)$$

$$= 15.1$$

11. 22.6 cm

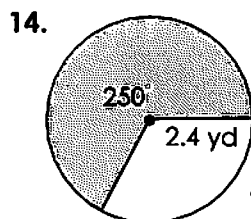
12. 15.1 ft

Find the area of each sector. Round your answer to the nearest tenth.



$$A = \frac{1}{2} \left(\frac{\pi}{5} \right) (17)^2$$

$$= \frac{\pi}{10} (289) \approx 90.8$$



$$250^\circ \rightarrow \frac{25\pi}{18}$$

$$A = \frac{1}{2} \left(\frac{25\pi}{18} \right) (2.4)^2$$

$$= \frac{25\pi}{36} (5.76) \approx 12.6$$

13. 90.8 in²

14. 12.6 yd²

Use your knowledge of arc lengths, area of sectors, and circular motion to solve each problem.

15. Lincoln, Nebraska lies directly north of Dallas, Texas. Lincoln is at a latitude of 40.5° N and Dallas is at a latitude of 32.5° N. Assuming the radius of the earth is 3,960 miles, how far apart are these cities?

$$8^\circ \rightarrow \frac{2\pi}{45}$$

$$S = 3960 \left(\frac{2\pi}{45} \right) \approx 552.9$$

15. 552.9 mi

16. 572.7 ft²

16. A motion detector can detect an object up to 25 feet away through an angle of 105°. What is the area of the region monitored?

$$105^\circ \rightarrow \frac{7\pi}{12}$$

$$A = \frac{1}{2} (25)^2 \left(\frac{7\pi}{12} \right)$$

$$= \frac{7\pi}{24} (625)$$

$$\approx 572.7$$

17. 76.2 mi/hr

18. 125.7 rad/sec

17. The diameter of each tire on a vehicle is 32 inches. If the tires are moving at a rate of 800 revolutions per minute, find the linear speed of the vehicle in miles per hour.

$$S = 2\pi r \cdot \text{rev/min} = 25600\pi \text{ in}; V = \frac{25600\pi \text{ in}}{1 \text{ min}}$$

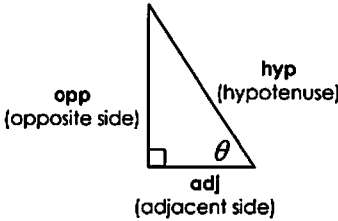
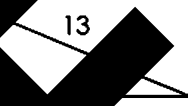
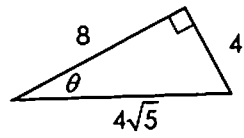
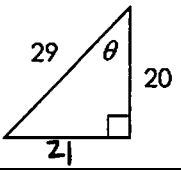
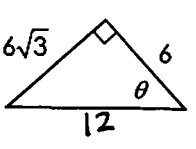
$$\frac{25600\pi \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 76.2 \text{ mi/hr}$$

18. The drum in a washing machine spins at 1,200 revolutions per minute. If the diameter of the drum is 76 centimeters, find the angular speed of the drum in radians per second.

$$\omega = 2\pi (1200) = 2400\pi \text{ rad}; W = \frac{2400\pi \text{ rad}}{1 \text{ min}}$$

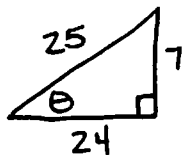
$$\frac{2400\pi \text{ rad}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx 125.7 \text{ rad/sec}$$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples			
TRIGONOMETRIC FUNCTIONS 	Trigonometry is the study of the relationships among the sides and angles and measures in triangles. - A trigonometric ratio compares the lengths of two sides of a right triangle with an acute angle, θ (theta), in a right triangle. - A trigonometric function is a rule defined by the trigonometric ratios.			
	Use the triangle to the left to define the trigonometric functions.			
	SINE	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	COSINE	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$
	COSINE	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$	SINE	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$
	TANGENT	$\tan \theta = \frac{\text{opp}}{\text{adj}}$	COTANGENT	$\cot \theta = \frac{\text{adj}}{\text{opp}}$
	Because the cosecant, secant, and cotangent ratios are reciprocals of the sine, cosine, and tangent ratios, they are called reciprocal functions . They can be defined as:			
	COTANGENT	$\cot \theta = \frac{1}{\tan \theta}$	SECANT	$\sec \theta = \frac{1}{\cos \theta}$
	COSINE	$\cos \theta = \frac{1}{\sec \theta}$	COTANGENT	$\cot \theta = \frac{1}{\tan \theta}$
EXAMPLES	Find the exact values of the six trigonometric functions of θ .			
	1.		2.	
	$\sin \theta = \frac{5}{13}$	$\csc \theta = \frac{13}{5}$	$\sin \theta = \frac{4}{8} = \frac{\sqrt{5}}{2}$	$\csc \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
	$\cos \theta = \frac{12}{13}$	$\sec \theta = \frac{13}{12}$	$\cos \theta = \frac{4\sqrt{5}}{8} = \frac{\sqrt{5}}{2}$	$\sec \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
	$\tan \theta = \frac{5}{12}$	$\cot \theta = \frac{12}{5}$	$\tan \theta = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$	$\cot \theta = \frac{4\sqrt{5}}{4} = \sqrt{5}$
	3.		4.	
	$\sin \theta = \frac{20}{29}$	$\csc \theta = \frac{29}{20}$	$\sin \theta = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}}$	$\csc \theta = \frac{\sqrt{3}}{1} = \sqrt{3}$
	$\cos \theta = \frac{21}{29}$	$\sec \theta = \frac{29}{21}$	$\cos \theta = \frac{12}{6\sqrt{3}} = \frac{2}{\sqrt{3}}$	$\sec \theta = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$
	$\tan \theta = \frac{20}{21}$	$\cot \theta = \frac{21}{20}$	$\tan \theta = \frac{6}{12} = \frac{1}{2}$	$\cot \theta = \frac{12}{6} = 2$
			$\tan \theta = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$	$\cot \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
			$\tan \theta = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$	$\cot \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
			$\tan \theta = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$	$\cot \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Use the given trigonometric value to find the remaining function values.

5. $\tan \theta = \frac{7}{24}$

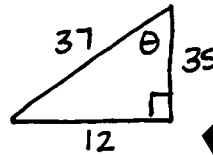


$$7^2 + 24^2 = c^2$$

$$625 = c^2$$

$$25 = c$$

6. $\csc \theta = \frac{37}{12} \rightarrow \sin \theta = \frac{12}{37}$



$$a^2 + 12^2 = 37^2$$

$$a^2 = 1225$$

$$a = 35$$

$\sin \theta = \frac{7}{25}$

$\csc \theta = \frac{25}{7}$

$\sin \theta = \frac{12}{37}$

$\csc \theta = \frac{37}{12}$

$\cos \theta = \frac{24}{25}$

$\sec \theta = \frac{25}{24}$

$\cos \theta = \frac{35}{37}$

$\sec \theta = \frac{37}{35}$

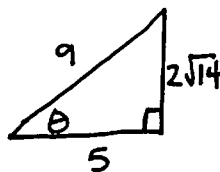
$\tan \theta = \frac{7}{24}$

$\cot \theta = \frac{24}{7}$

$\tan \theta = \frac{12}{35}$

$\cot \theta = \frac{35}{12}$

7. $\cos \theta = \frac{5}{9}$



$$5^2 + b^2 = 9^2$$

$$b^2 = 56$$

$$b = 2\sqrt{14}$$

8. $\cos \theta = \frac{4}{5} \rightarrow \cos \theta = \frac{4}{5}$



$$1^2 + b^2 = 4^2$$

$$b^2 = 15$$

$$b = \sqrt{15}$$

$\sin \theta = \frac{2\sqrt{14}}{9}$

$\csc \theta = \frac{9}{2\sqrt{14}} = \frac{9\sqrt{14}}{28}$

$\sin \theta = \frac{\sqrt{15}}{4}$

$\csc \theta = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$

$\cot \theta = \frac{5}{9}$

$\tan \theta = \frac{2\sqrt{14}}{5}$

$\cos \theta = \frac{4}{5}$

$\sec \theta = \frac{5}{4}$

$\theta = \arccos\left(\frac{5}{9}\right)$

$\cot \theta = \frac{5}{2\sqrt{14}} = \frac{5\sqrt{14}}{28}$

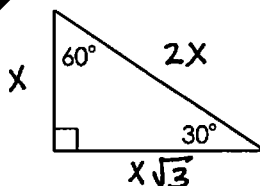
$\tan \theta = \frac{\sqrt{15}}{4}$

$\cot \theta = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$

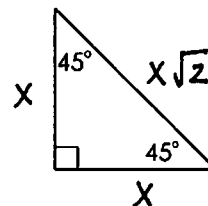
TRIG VALUES SPECIAL ANGLES

Angles of 30° , 45° , and 60° are used frequently in trigonometry. For your knowledge of the side relationships in special right triangles, we will find the values of the trigonometric ratios.

30° - 60° - 90° Triangle



45° - 45° - 90° Triangle



θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Name: _____

Unit 5: Trigonometric Functions

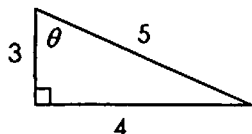
Date: _____ Per: _____

Homework 4: Trigonometric Functions

** This is a 2-page document! **

Directions: Find the exact values of the trigonometric functions of θ .

1.



$\sin \theta = \frac{4}{5}$

$\csc \theta = \frac{5}{4}$

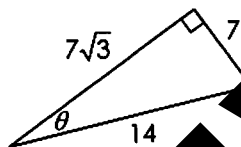
$\cos \theta = \frac{3}{5}$

$\sec \theta = \frac{5}{3}$

$\tan \theta = \frac{4}{3}$

$\cot \theta = \frac{3}{4}$

2.



$\sin \theta = \frac{7}{14} = \frac{1}{2}$

$\csc \theta = \frac{14}{7} = 2$

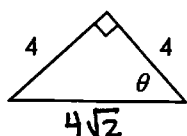
$\cos \theta = \frac{7\sqrt{3}}{14} = \frac{\sqrt{3}}{2}$

$\sec \theta = \frac{14}{7\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan \theta = \frac{7}{7\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\cot \theta = \frac{7\sqrt{3}}{7} = \sqrt{3}$

3.



$4^2 + 4^2 = c^2$

$32 = c^2$

$c = \sqrt{32} = 4\sqrt{2}$

$\sin \theta = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\csc \theta = \frac{4\sqrt{2}}{4} = \sqrt{2}$

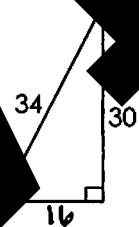
$\cos \theta = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sec \theta = \frac{4\sqrt{2}}{4} = \sqrt{2}$

$\tan \theta = \frac{4}{4} = 1$

$\cot \theta = \frac{4}{4} = 1$

4.



$a^2 + 30^2 = 34^2$

$a^2 = 256$

$a = 16$

$\sin \theta = \frac{30}{34} = \frac{15}{17}$

$\csc \theta = \frac{34}{30} = \frac{17}{15}$

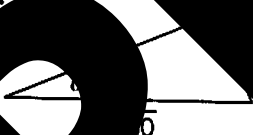
$\cos \theta = \frac{16}{34} = \frac{8}{17}$

$\sec \theta = \frac{34}{16} = \frac{17}{8}$

$\tan \theta = \frac{30}{16} = \frac{15}{8}$

$\cot \theta = \frac{16}{30} = \frac{8}{15}$

5.



$5^2 + b^2 = (\sqrt{30})^2$

$b^2 = 5$

$b = \sqrt{5}$

$\sin \theta = \frac{5}{\sqrt{30}} = \frac{\sqrt{6}}{6}$

$\csc \theta = \frac{\sqrt{30}}{5} = \sqrt{6}$

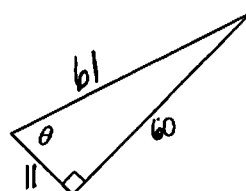
$\cos \theta = \frac{5}{\sqrt{30}} = \frac{\sqrt{30}}{6}$

$\sec \theta = \frac{\sqrt{30}}{5}$

$\tan \theta = \frac{\sqrt{5}}{5}$

$\cot \theta = \frac{5}{\sqrt{5}} = \sqrt{5}$

6.



$11^2 + b^2 = c^2$

$3721 = c^2$

$c = 61$

$\sin \theta = \frac{11}{61}$

$\csc \theta = \frac{61}{11}$

$\cos \theta = \frac{11}{61}$

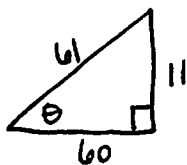
$\sec \theta = \frac{61}{11}$

$\tan \theta = \frac{11}{60}$

$\cot \theta = \frac{60}{11}$

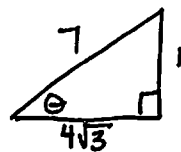
Directions: Use the given trigonometric function value of θ to find the exact values of the remaining functions.

7. $\cos \theta = \frac{60}{61}$ $a^2 + b^2 = c^2$



$a^2 = 121$
 $a = 11$

8. $\csc \theta = 7$



$1^2 + b^2 = 7^2$
 $b^2 = 48$
 $b = 4\sqrt{3}$

$\sin \theta = \frac{11}{61}$

$\csc \theta = \frac{61}{11}$

$\sin \theta = \frac{1}{7}$

$\csc \theta = 7$

$\cos \theta = \frac{60}{61}$

$\sec \theta = \frac{61}{60}$

$\cos \theta = \frac{4\sqrt{3}}{7}$

$\sec \theta = \frac{7\sqrt{3}}{12}$

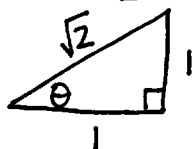
$\tan \theta = \frac{11}{60}$

$\cot \theta = \frac{60}{11}$

$\tan \theta = \frac{\sqrt{3}}{4}$

$\cot \theta = \frac{4\sqrt{3}}{1} = 4\sqrt{3}$

9. $\sin \theta = \frac{\sqrt{2}}{2}$



$1^2 + b^2 = (\sqrt{2})^2$
 $b^2 = 1$
 $b = 1$

10. $\theta = \frac{2}{3}$



$3^2 = c^2$
 $13 = c^2$
 $c = \sqrt{13}$

$\sin \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\csc \theta = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\csc \theta = \frac{\sqrt{13}}{2}$

$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sec \theta = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\sec \theta = \frac{\sqrt{13}}{3}$

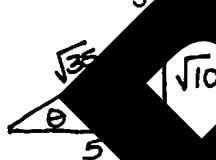
$\tan \theta = \frac{1}{1} = 1$

$\theta = 1$

$\tan \theta = \frac{2}{3}$

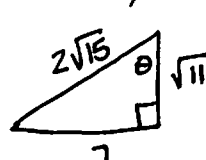
$\cot \theta = \frac{3}{2}$

11. $\sec \theta = \frac{\sqrt{35}}{5}$



$5^2 = c^2$
 $25 = c^2$
 $b = \sqrt{10}$

12. $\cot \theta = \frac{\sqrt{11}}{7}$



$7^2 + (\sqrt{11})^2 = c^2$
 $60 = c^2$
 $2\sqrt{15} = c$

$\sin \theta = \frac{\sqrt{10}}{5}$

$\csc \theta = \frac{\sqrt{35}}{\sqrt{10}} = \frac{\sqrt{14}}{2}$

$\sin \theta = \frac{7}{2\sqrt{15}} = \frac{7\sqrt{15}}{30}$

$\csc \theta = \frac{2\sqrt{15}}{7}$

$\cos \theta = \frac{\sqrt{35}}{5}$

$\sec \theta = \frac{\sqrt{35}}{5}$

$\cos \theta = \frac{\sqrt{11}}{2\sqrt{15}} = \frac{\sqrt{165}}{30}$

$\sec \theta = \frac{2\sqrt{15}}{\sqrt{11}} = \frac{2\sqrt{165}}{11}$

$\tan \theta = \frac{\sqrt{10}}{5}$

$\cot \theta = \frac{5}{\sqrt{10}} = \frac{\sqrt{10}}{2}$

$\tan \theta = \frac{7}{\sqrt{11}} = \frac{7\sqrt{11}}{11}$

$\cot \theta = \frac{\sqrt{11}}{7}$

Directions: Give the exact value of each function.

13. $\cos 60^\circ$

$\frac{1}{2}$

14. $\tan 45^\circ$

1

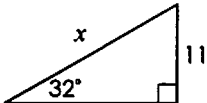
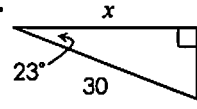
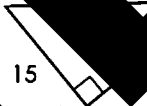
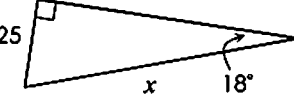
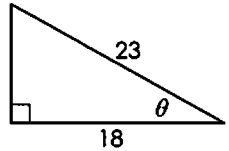
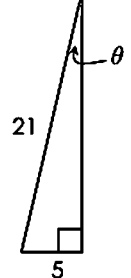
15. $\csc 45^\circ$

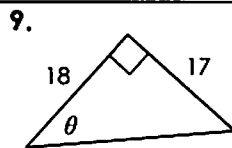
$\sqrt{2}$

16. $\cot 30^\circ$

$\sqrt{3}$

Name:	Date:
Topic:	Class:

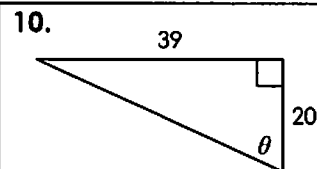
Main Ideas/Questions	Notes/Examples			
<i>Finding</i> SIDE LENGTHS	Find the value of x. Round to the nearest tenth. 1.  $\sin 32 = \frac{11}{x}$ $x = \frac{11}{\sin 32}$ $x = 20.8$ 2.  $x = \frac{21}{\cos 75}$ $x = 41.3$ 3.  $\cos 23 = \frac{x}{30}$ $x = 30 \cdot \cos 23$ $x = 27.7$ 4.  $\sin 41 = \frac{15}{x}$ $x = \frac{15}{\sin 41}$ $x = 17.3$ 5.  $\cos 64 = \frac{x}{3}$ $x = 3 \cdot \cos 64$ $x = 0.8$ 6.  $\sin 18 = \frac{25}{x}$ $x = \frac{25}{\sin 18}$ $x = 80.9$			
<i>Finding</i> ANGLE MEASURES	Given the trigonometric value of an angle, use the corresponding inverse trigonometric function to find the measure of the angle. <table><tr><td>If $\sin \theta = x$, then $\theta = \sin^{-1} x$</td><td>If $\cos \theta = x$, then $\theta = \cos^{-1} x$</td><td>If $\tan \theta = x$, then $\theta = \tan^{-1} x$</td></tr></table> Find the measure of θ. Round to the nearest tenth of a degree. 7.  $\cos \theta = \frac{18}{23}$ $\theta = \cos^{-1} \left(\frac{18}{23} \right)$ $\theta = 38.5^\circ$ 8.  $\sin \theta = \frac{5}{21}$ $\theta = \sin^{-1} \left(\frac{5}{21} \right)$ $\theta = 13.8^\circ$	If $\sin \theta = x$, then $\theta = \sin^{-1} x$	If $\cos \theta = x$, then $\theta = \cos^{-1} x$	If $\tan \theta = x$, then $\theta = \tan^{-1} x$
If $\sin \theta = x$, then $\theta = \sin^{-1} x$	If $\cos \theta = x$, then $\theta = \cos^{-1} x$	If $\tan \theta = x$, then $\theta = \tan^{-1} x$		



$$\tan \theta = \frac{17}{18}$$

$$\theta = \tan^{-1}\left(\frac{17}{18}\right)$$

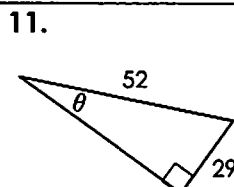
$$\theta = 43.4^\circ$$



$$\tan \theta = \frac{39}{20}$$

$$\theta = \tan^{-1}\left(\frac{39}{20}\right)$$

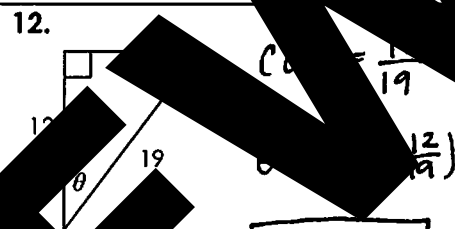
$$\theta = 62.5^\circ$$



$$\sin \theta = \frac{29}{52}$$

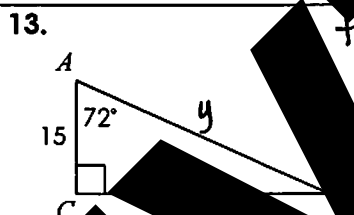
$$\theta = \sin^{-1}\left(\frac{29}{52}\right)$$

$$\theta = 33.0^\circ$$



$$\theta = 50.8^\circ$$

Solve each triangle. Round measures to the nearest tenth.



$$\tan 72^\circ = \frac{15}{y}$$

$$y = 46.2$$

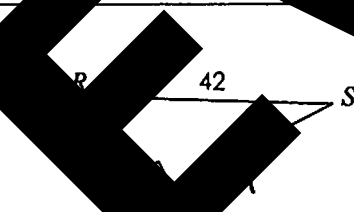
$$\cos 72^\circ = \frac{15}{y}$$

$$y = 48.5$$

$$BC = 46.2$$

$$AB = 48.5$$

$$m\angle B = 18^\circ$$



$$\sin 34^\circ = \frac{y}{42}$$

$$y = 23.5$$

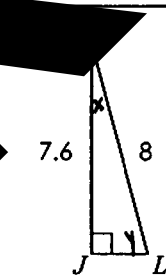
$$\cos 34^\circ = \frac{y}{42}$$

$$y = 34.8$$

$$RT = 23.5$$

$$ST = 34.8$$

$$m\angle R = 56^\circ$$



$$\cos x = \frac{7.6}{8}$$

$$x = 18.2^\circ$$

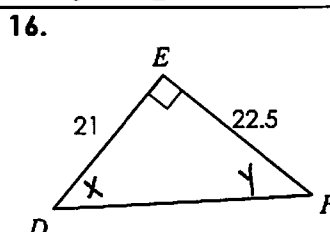
$$\sin x = \frac{7.6}{8}$$

$$x = 71.8^\circ$$

$$JL = 2.5$$

$$m\angle K = 18.2^\circ$$

$$m\angle L = 71.8^\circ$$



$$\tan x = \frac{22.5}{21}$$

$$x = 47^\circ$$

$$\tan y = \frac{21}{22.5}$$

$$y = 43^\circ$$

$$DF = 30.8$$

$$m\angle D = 47^\circ$$

$$m\angle F = 43^\circ$$

Solving RIGHT TRIANGLES

Solving a right triangle means to find all side and angle measures.

Name:

Date:

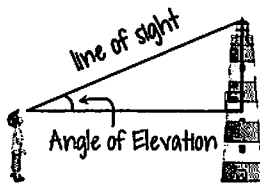
Topic:

Class:

Main Ideas/Questions

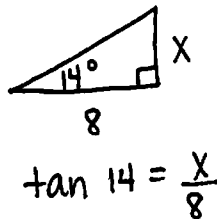
Notes/Examples

ANGLE OF ELEVATION



Round all answers to the nearest tenth when necessary.

1. A fire started at a campsite located 8 miles from the base of a mountain. If the angle of elevation from the fire to the top of the mountain is 14° , how high is the mountain?



$$X = 2 \text{ miles}$$

2. A man is standing 200 meters from the base of a tower. The angle of elevation from the man's eye level to the top of the tower is 67° . How tall is the tower?

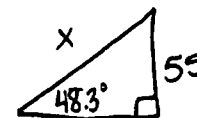


$$X = 370.5 \text{ m}$$

3. A 32-foot tall ladder is leaning against a vertical wall. If the top of the ladder reaches a point 25 feet up the wall, how far is the bottom of the ladder from the wall?



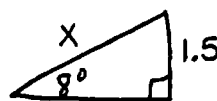
4. A kite is flying 60 feet above the ground. The angle of elevation from where her hand is holding the string to the top of the kite is 48.3° . If her hand is 5 feet from the ground, find the length of the string.



$$\sin 48.3 = \frac{55}{X}$$

$$X = 73.7 \text{ ft}$$

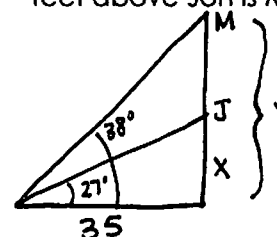
5. A ramp is being built to access a door that is 1.5 feet above the ground. Find the length of the ramp if the angle of elevation must be 8° .



$$\sin 8 = \frac{1.5}{X}$$

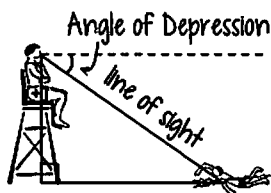
$$X = 10.8 \text{ ft}$$

6. Sara is standing 35 feet from the base of a rock wall watching her two sons, Max and Jon, climb the wall. The angle of elevation is 38° to Max and 27° to Jon. How many feet above Jon is Max?

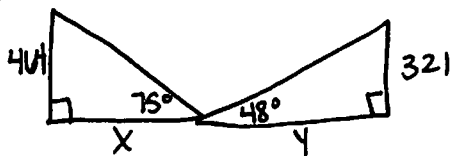


$$\text{Max is } 9.5 \text{ ft higher.}$$

ANGLE OF DEPRESSION



7. Building A is 464 feet tall and Building B is 321 feet tall. Dale is standing between the buildings. The angle of elevation from the point on the ground where Dale is standing is 75° to the top of Building A and 48° to top of Building B. How far apart are the buildings?



$$\tan 75 = \frac{464}{X}$$

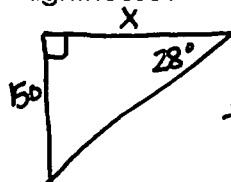
$$X = 124.3 \text{ ft}$$

$$\tan 48 = \frac{321}{Y}$$

$$Y = 289.1 \text{ ft}$$

$$413.3 \text{ ft apart}$$

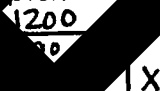
8. From the top of a lighthouse 150 feet above sea level, the angle of depression to a boat at sea is 28° . What is the horizontal distance from the boat to the base of the lighthouse?



$$\tan 28 = \frac{150}{X}$$

$$X = 321.1 \text{ ft}$$

9. The angle of depression from a helicopter to a landing pad is 34° . The horizontal distance from the helicopter to the landing pad is 1200 feet. Find the altitude of the helicopter.



$$\tan 34 = \frac{X}{1200}$$

$$X = 837.1 \text{ ft}$$

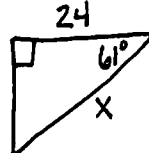
10. Henry is standing on a tree 32 feet above the ground. A deer is 57 feet from the base of the tree. Find the angle of depression from Henry to the deer.



$$\tan \theta = \frac{32}{57}$$

$$\theta = 29.3^\circ$$

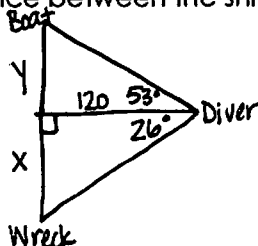
11. A wire is attached from the top of a post to a point on the ground 24 feet from the base of the post. If the angle of depression from the top of the post to the point where the wire is attached to the ground is 61° , find the length of the wire.



$$\cos 61 = \frac{24}{X}$$

$$X = 49.5 \text{ ft}$$

12. A scuba diver spots a shipwreck at an angle of depression of 26° . The angle of elevation from the scuba diver up to his boat is 53° . If the shipwreck is located directly below his boat, and the horizontal distance between the scuba diver and the shipwreck is 120 feet, find the vertical distance between the shipwreck and the boat.



$$\tan 26 = \frac{X}{120}$$

$$X = 58.5 \text{ ft}$$

$$\tan 53 = \frac{Y}{120}$$

$$Y = 159.2 \text{ ft}$$

$$\text{Vertical Distance} = 217.7 \text{ ft}$$

Name: _____

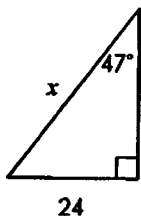
Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 5: Solving Right Triangles

**** This is a 2-page document! ******Directions:** Find the missing side or angle measure. Round answers to the nearest tenth.

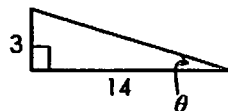
1.



$$\sin 47 = \frac{24}{x}$$

$$x = \frac{24}{\sin 47}$$

$$x = 32.8$$



$$\tan \theta = \frac{3}{14}$$

$$\theta = \tan^{-1}\left(\frac{3}{14}\right)$$

$$\theta = 12.1^\circ$$

3.

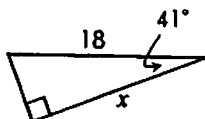


$$\cos \theta = \frac{39}{45}$$

$$\theta = \cos^{-1}\left(\frac{39}{45}\right)$$

$$\theta = 29.9^\circ$$

4.



$$\cos 41 = \frac{x}{18}$$

$$x = 13.6$$

5.

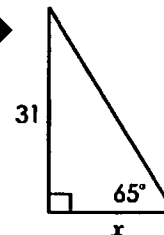


$$\sin \theta = \frac{4}{7}$$

$$\theta = \sin^{-1}\left(\frac{4}{7}\right)$$

$$\theta = 34.8^\circ$$

6.



$$\tan 65 = \frac{31}{x}$$

$$x = \frac{31}{\tan 65}$$

$$x = 14.5$$

Directions: Solve each triangle. Round all measures to the nearest tenth.

7.



$$\tan 50 = \frac{14}{x}$$

$$x = 11.7$$

$$\sin 50 = \frac{14}{y}$$

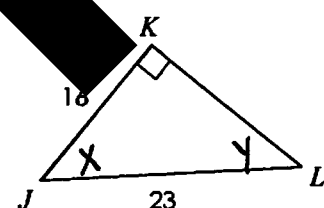
$$y = 18.3$$

$$AB = 11.7$$

$$AC = 18.3$$

$$m\angle C = 40^\circ$$

8.



$$\cos x = \frac{16}{23}$$

$$x = 45.9^\circ$$

$$\sin y = \frac{16}{23}$$

$$y = 44.1^\circ$$

$$KL = 16.5$$

$$m\angle J = 45.9^\circ$$

$$m\angle L = 44.1^\circ$$

9.



$$\tan x = \frac{4.6}{2}$$

$$x = 66.5^\circ$$

$$\tan y = \frac{2}{4.6}$$

$$y = 23.5^\circ$$

$$FG = 5$$

$$m\angle F = 23$$

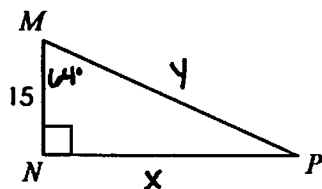
$$m\angle G = 66.5$$

$$MP = 4.6$$

$$m\angle P = 23$$

$$m\angle P = 26$$

10.



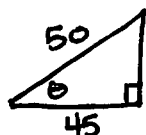
$$\cos 64 = \frac{15}{y}$$

$$y = 34.2$$

$$\tan 64 = \frac{x}{15}$$

$$x = 30.1$$

11. A 50-foot ladder is leaning against a vertical wall. If the base of the ladder is 45 feet from the base of the wall, find the angle the bottom of the ladder makes with the ground.



$$\cos \theta = \frac{45}{50}$$

$$\theta = \cos^{-1} \left(\frac{45}{50} \right)$$

$$\theta = 25.8^\circ$$

12. Joel is zip lining over the mountains. If the vertical distance from his platform to the ground is 21.725 feet, and the angle of depression is 21.725° , find the length of the rope.

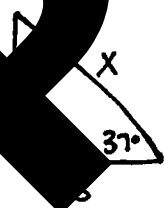


$$\sin 21.725 = \frac{21.725}{x}$$

$$x = \frac{21.725}{\sin 21.725}$$

$$x = 821.3 \text{ yd}$$

13. A string is connected to the corner of a square canopy and a point on the ground 15 feet from the corner. The angle of elevation from the point on the ground to the corner of the canopy is 37° , find the length of the string.

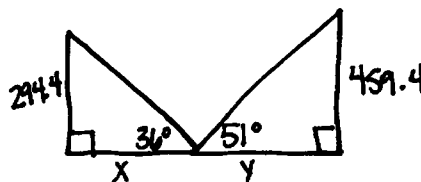


$$\cos 32 = \frac{15}{x}$$

$$x = \frac{15}{\cos 32}$$

$$x = 18.8 \text{ ft}$$

14. At a hot air balloon festival, Erica spots a red balloon due east at an angle of elevation of 36° . She turns due west and sees a blue balloon at an angle of elevation of 51° . If altitude of the red balloon is 296 meters, the altitude of the blue balloon is 461 meters, and Erica is 1.6 meters tall, find the distance between the balloons.



$$\tan 36 = \frac{294.4}{x}$$

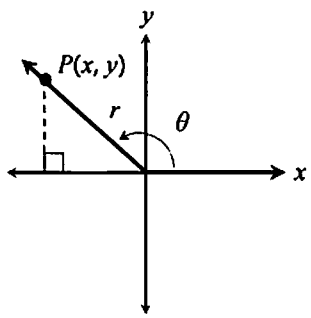
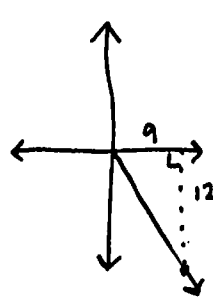
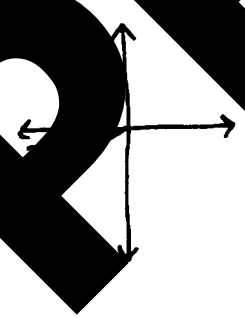
$$x = 405.2$$

$$\tan 51 = \frac{459.4}{y}$$

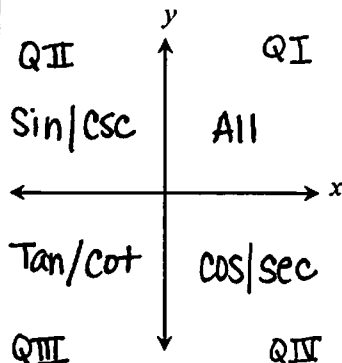
$$y = 372$$

$$777.2 \text{ m}$$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples						
TRIG FUNCTIONS for any angle   	In the previous lessons, you found trigonometric functions for positive acute angles. We will now find trigonometric functions for any angle. Let θ be an angle in standard form and $P(x, y)$ be a point on the terminal side of θ. The distance from P to the origin is the length r. r can be found using the formula: $x^2 + y^2 = r^2$ (Pythagorean Theorem)						
	<table><tr><td>$\sin \theta = \frac{y}{r}$</td><td>$\cos \theta = \frac{x}{r}$</td><td>$\tan \theta = \frac{y}{x}$</td></tr><tr><td>$\csc \theta = \frac{r}{y}$</td><td>$\sec \theta = \frac{r}{x}$</td><td>$\cot \theta = \frac{x}{y}$</td></tr></table>	$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$	$\csc \theta = \frac{r}{y}$	$\sec \theta = \frac{r}{x}$	$\cot \theta = \frac{x}{y}$
	$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$				
	$\csc \theta = \frac{r}{y}$	$\sec \theta = \frac{r}{x}$	$\cot \theta = \frac{x}{y}$				
	1. $P(9, -12)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ. $9^2 + 12^2 = r^2$ $225 = r^2$ $15 = r$ $y = -12$ $r = 15$						
<table><tr><td>$\sin \theta = \frac{-12}{15} = -\frac{4}{5}$</td><td>$\cos \theta = \frac{9}{15} = \frac{3}{5}$</td><td>$\tan \theta = \frac{-12}{9} = -\frac{4}{3}$</td></tr><tr><td>$\csc \theta = \frac{15}{-12} = -\frac{5}{4}$</td><td>$\sec \theta = \frac{15}{9} = \frac{5}{3}$</td><td>$\cot \theta = \frac{9}{-12} = -\frac{3}{4}$</td></tr></table>	$\sin \theta = \frac{-12}{15} = -\frac{4}{5}$	$\cos \theta = \frac{9}{15} = \frac{3}{5}$	$\tan \theta = \frac{-12}{9} = -\frac{4}{3}$	$\csc \theta = \frac{15}{-12} = -\frac{5}{4}$	$\sec \theta = \frac{15}{9} = \frac{5}{3}$	$\cot \theta = \frac{9}{-12} = -\frac{3}{4}$	
$\sin \theta = \frac{-12}{15} = -\frac{4}{5}$	$\cos \theta = \frac{9}{15} = \frac{3}{5}$	$\tan \theta = \frac{-12}{9} = -\frac{4}{3}$					
$\csc \theta = \frac{15}{-12} = -\frac{5}{4}$	$\sec \theta = \frac{15}{9} = \frac{5}{3}$	$\cot \theta = \frac{9}{-12} = -\frac{3}{4}$					
2. $P(-6, -2)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ. $2^2 + 6^2 = r^2$ $40 = r^2$ $2\sqrt{10} = r$ $x = -6$ $y = -2$ $r = 2\sqrt{10}$							
<table><tr><td>$\sin \theta = \frac{-2}{2\sqrt{10}} = -\frac{\sqrt{10}}{10}$</td><td>$\cos \theta = \frac{-6}{2\sqrt{10}} = -\frac{3\sqrt{10}}{10}$</td><td>$\tan \theta = \frac{-2}{-6} = \frac{1}{3}$</td></tr><tr><td>$\csc \theta = \frac{2\sqrt{10}}{-2} = -\sqrt{10}$</td><td>$\sec \theta = \frac{2\sqrt{10}}{-6} = -\frac{\sqrt{10}}{3}$</td><td>$\cot \theta = \frac{-6}{-2} = 3$</td></tr></table>	$\sin \theta = \frac{-2}{2\sqrt{10}} = -\frac{\sqrt{10}}{10}$	$\cos \theta = \frac{-6}{2\sqrt{10}} = -\frac{3\sqrt{10}}{10}$	$\tan \theta = \frac{-2}{-6} = \frac{1}{3}$	$\csc \theta = \frac{2\sqrt{10}}{-2} = -\sqrt{10}$	$\sec \theta = \frac{2\sqrt{10}}{-6} = -\frac{\sqrt{10}}{3}$	$\cot \theta = \frac{-6}{-2} = 3$	
$\sin \theta = \frac{-2}{2\sqrt{10}} = -\frac{\sqrt{10}}{10}$	$\cos \theta = \frac{-6}{2\sqrt{10}} = -\frac{3\sqrt{10}}{10}$	$\tan \theta = \frac{-2}{-6} = \frac{1}{3}$					
$\csc \theta = \frac{2\sqrt{10}}{-2} = -\sqrt{10}$	$\sec \theta = \frac{2\sqrt{10}}{-6} = -\frac{\sqrt{10}}{3}$	$\cot \theta = \frac{-6}{-2} = 3$					

SIGNS OF TRIG FUNCTIONS



Depending on the location of the terminal side of the angle, trigonometric functions can be positive or negative. Using your knowledge of the trigonometric function values, identify the functions that are positive in each quadrant.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
All	Sin/csc	Tan /Cot	Cos/sec
Remember the phrase "All Students Take Calculus"			
3. If $\cos \theta > 0$, which quadrant(s) could the terminal side of θ lie? Quad I, IV		4. If $\tan \theta < 0$, which quadrant(s) could the terminal side of θ lie? Quad II, I	
5. If $\sec \theta < 0$, which quadrant(s) could the terminal side of θ lie? Quad II, III		6. If $\sin \theta > 0$ and $\cot \theta > 0$, which quadrant(s) could the terminal side of θ lie? Quad III	

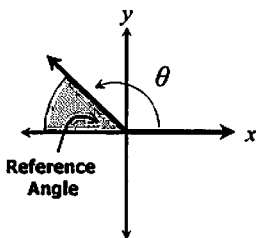
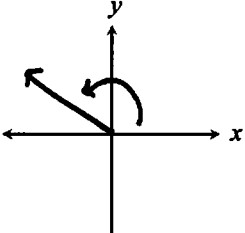
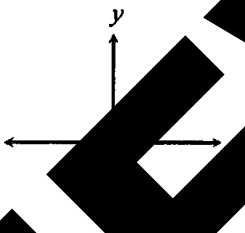
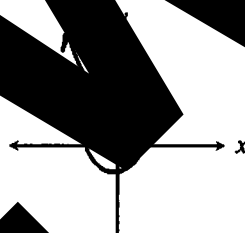
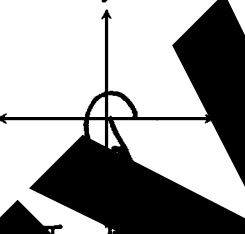
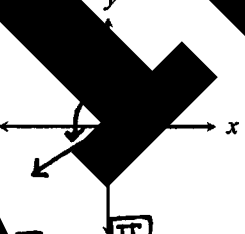
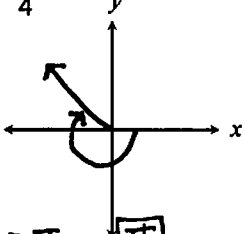
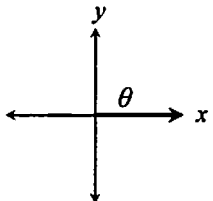
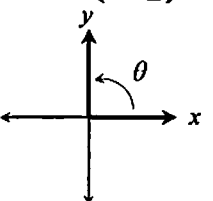
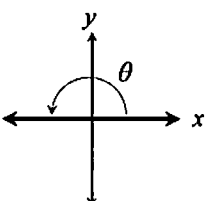
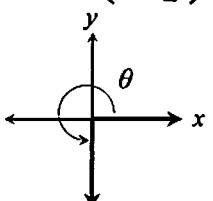
USING TRIG VALUES

to find other values

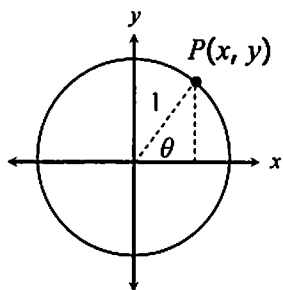
Use the given information to find the exact values of the five remaining trigonometric functions of θ .

7. $\sin \theta = \frac{8}{17}$; $\tan \theta < 0$ (Quad II)	$\cos \theta = -\frac{15}{17}$
	$\tan \theta = -\frac{8}{15}$
	$\csc \theta = \frac{17}{8}$
	$\sec \theta = -\frac{17}{15}$
	$\cot \theta = -\frac{15}{8}$
8. $\tan \theta = -\frac{1}{4}$; $\sec \theta > 0$ (Quad IV)	$\sin \theta = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$
	$\cos \theta = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$
	$\tan \theta = -\frac{1}{4}$
	$\csc \theta = -\frac{\sqrt{17}}{1} = -\sqrt{17}$
	$\sec \theta = \frac{\sqrt{17}}{4}$
9. $\csc \theta = -\frac{\sqrt{5}}{2}$; $\cos \theta < 0$ (Quad III)	$\sin \theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$
	$\cos \theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$
	$\tan \theta = \frac{2}{1} = 2$
	$\sec \theta = \frac{\sqrt{5}}{-1} = -\sqrt{5}$
	$\cot \theta = \frac{1}{2}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples			
REFERENCE ANGLES 	For an angle θ in standard form, the reference angle is the positive acute angle form by the terminal side of θ to the x-axis. Sketch each angle, then find its reference angle.			
	1. 150°  $180 - 150 = \boxed{30^\circ}$	2. 315°  $360 - 315 = \boxed{45^\circ}$	3. -240°  $-180 - (-240) = \boxed{60^\circ}$	
	4. $\frac{5\pi}{3} \rightarrow 300^\circ$  $360 - 300 = \boxed{60^\circ}$ $2\pi - \frac{5\pi}{3} = \boxed{\frac{\pi}{3}}$	5. 210°  $210 - 180 = \boxed{30^\circ}$ $\pi - \frac{7\pi}{6} = \boxed{\frac{\pi}{6}}$	6. $-\frac{5\pi}{4} \rightarrow -225^\circ$  $-180 - (-225) = \boxed{45^\circ}$ $\frac{5\pi}{4} - \pi = \boxed{\frac{\pi}{4}}$	
	Recall the signs of the functions in each quadrant! Use reference angles and trigonometric function values for angles in special right triangles to find each trigonometric value.			
	7. $\sec 150^\circ \rightarrow 60^\circ$ $\frac{\sqrt{3}}{2}$	8. $\sec 225^\circ \rightarrow 45^\circ$ $(\cos 45^\circ = \frac{\sqrt{2}}{2})$ $-\frac{2}{\frac{\sqrt{2}}{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$		
	9. $\frac{11\pi}{6} \rightarrow 30^\circ$ $-\frac{\sqrt{3}}{3}$	10. $\csc\left(-\frac{2\pi}{3}\right) \rightarrow -60^\circ$ $(\sin -60^\circ = -\frac{\sqrt{3}}{2})$ $-\frac{2}{-\frac{\sqrt{3}}{2}} = \boxed{\frac{2\sqrt{3}}{3}}$		
QUADRANTAL ANGLES	When the terminal side of an angle θ that is in standard position lies on the x- and y-axis, the angle is called a quadrantal angle .			
	$\theta = 0^\circ$ (or 0π) 	$\theta = 90^\circ$ (or $\frac{\pi}{2}$) 	$\theta = 180^\circ$ (or π) 	$\theta = 270^\circ$ (or $\frac{3\pi}{2}$) 

THE UNIT CIRCLE



A unit circle is a circle with a radius of 1 unit.
Because the value of r is 1 for each point $P(x, y)$ on the circle, the trigonometric functions of θ are defined as:

$$\sin \theta = y$$

$$\cos \theta = x$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{1}{y}$$

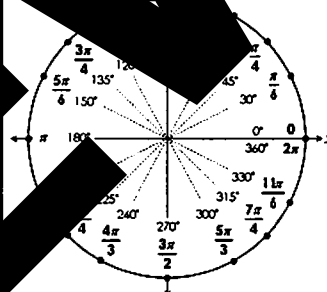
$$\sec \theta = \frac{1}{x}$$

$$\cot \theta = \frac{x}{y}$$

* The coordinates of P can be written as $(\cos \theta, \sin \theta)$

16-POINT UNIT CIRCLE

The unit circle is frequently used to find the values of trigonometric functions for angles $30^\circ, 45^\circ, 60^\circ$ (and their multiples) as well as $0^\circ/360^\circ, 90^\circ, 180^\circ, 270^\circ$ (the quadrantal angles).



THE UNIT CIRCLE & TRIG VALUES

Complete "The Unit Circle Reference Angles" and use the circle to find the exact value of each trigonometric function.

11. $\sin 135^\circ$

$$\frac{\sqrt{2}}{2}$$

12. $\sec 180^\circ$

$$(\cos 180^\circ = -1)$$

$$-1$$

13. $\cot(-300^\circ)$

$$\tan -300^\circ = \sqrt{3}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

14. $\sin 210^\circ$

$$-\frac{1}{2}$$

15. $\tan 270^\circ$

$$\text{undef}$$

16. $\cos(-150^\circ)$

$$-\frac{\sqrt{3}}{2}$$

17. $\cos \frac{3\pi}{4}$

$$-\frac{\sqrt{2}}{2}$$

18. $\tan \frac{2\pi}{3}$

$$-\sqrt{3}$$

19. $\cot \frac{\pi}{2}$

$$0$$

20. $\sec\left(-\frac{\pi}{4}\right)$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

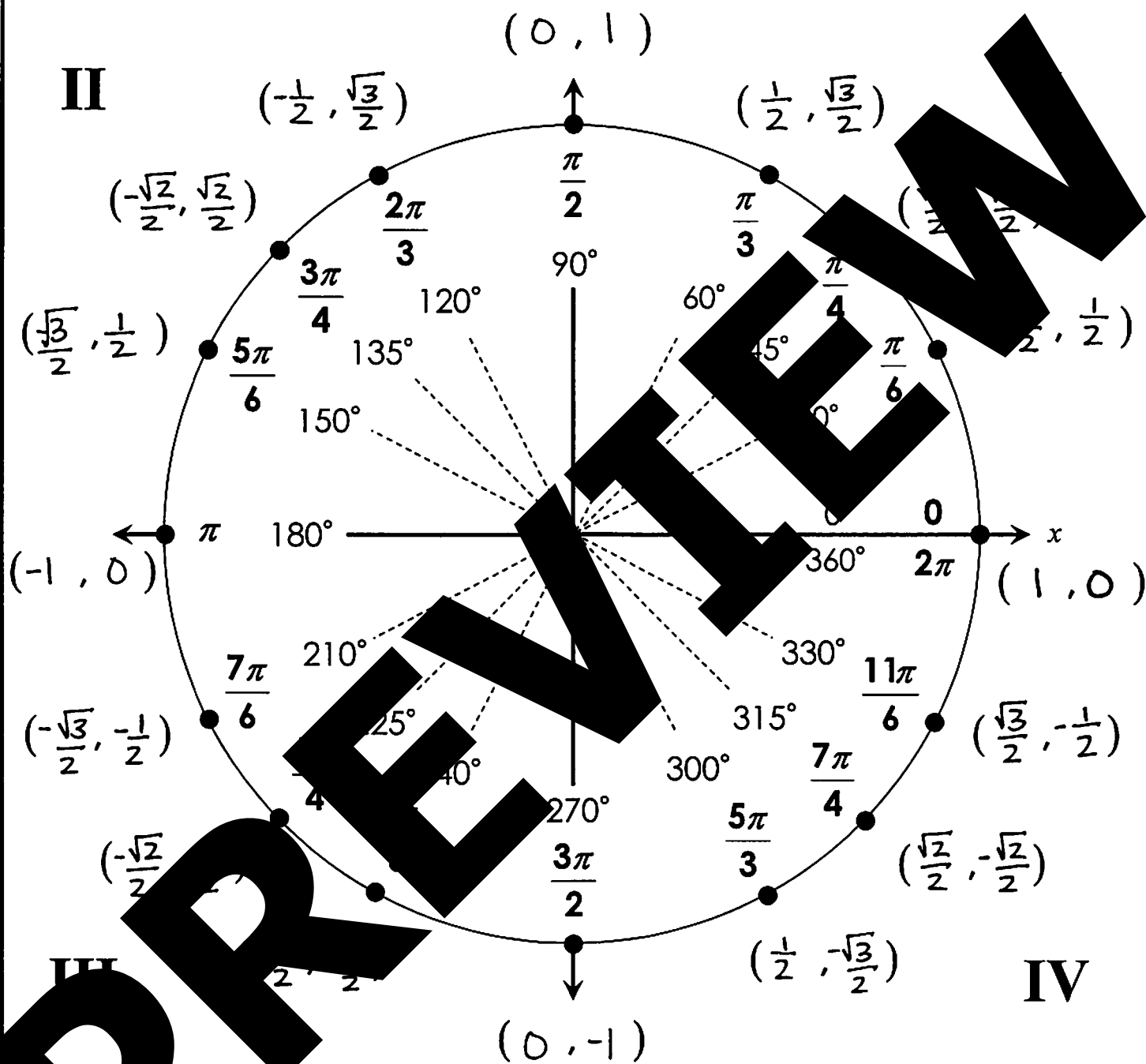
21. $\tan \frac{5\pi}{4}$

$$1$$

22. $\csc\left(-\frac{5\pi}{6}\right)$

$$-\frac{2}{1} = -2$$

THE UNIT CIRCLE *Reference!*



Trigonometric Values:

Quad I	All
Quad II	Sin, csc
Quad III	Tan, cot
Quad IV	cos, sec

Trigonometric Values:

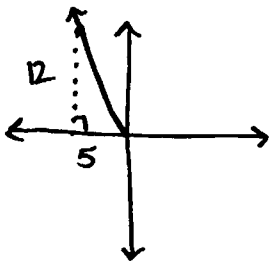
$\sin \theta = y$	$\csc \theta = \frac{1}{y}$
$\cos \theta = x$	$\sec \theta = \frac{1}{x}$
$\tan \theta = \frac{y}{x}$	$\cot \theta = \frac{x}{y}$

Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 6: Trig Functions of Any Angle & The Unit Circle

**** This is a 2-page document! ******Directions:** Given a point on the terminal side of θ in standard form, find the exact values of the five trigonometric functions of θ .1. $P(-5, 12)$ 

$$5^2 + 12^2 = r^2$$

$$169 = r^2$$

$$13 = r$$

$$x = -5$$

$$y = 12$$

$$r = 13$$

$$\sin \theta =$$

$$= \frac{12}{13}$$

$$\cos \theta =$$

$$= \frac{-5}{13}$$

$$\tan \theta =$$

$$= \frac{12}{-5}$$

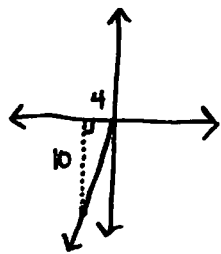
$$\csc \theta =$$

$$= \frac{13}{12}$$

$$\sec \theta =$$

$$= \frac{13}{-5}$$

$$\cot \theta =$$

2. $P(4, -10)$ 

$$4^2 + 10^2 = r^2$$

$$116 = r^2$$

$$2\sqrt{29} = r$$

$$x = 4$$

$$y = -10$$

$$r = 2\sqrt{29}$$

$$\sin \theta =$$

$$= \frac{-10}{2\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\cos \theta =$$

$$= \frac{4}{2\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\tan \theta =$$

$$= \frac{-10}{4} = -\frac{5}{2}$$

$$\csc \theta =$$

$$= \frac{2\sqrt{29}}{-10} = -\frac{\sqrt{29}}{5}$$

$$\sec \theta =$$

$$= \frac{2\sqrt{29}}{4} = \frac{\sqrt{29}}{2}$$

$$\cot \theta =$$

$$= \frac{-4}{-10} = \frac{2}{5}$$

Directions: Use the given information to find the exact values of the five remaining trigonometric functions of θ .3. $\tan \theta = \frac{21}{20}$; $\sin \theta < 0$; Quad IV

$$20^2 + 21^2 = c^2$$

$$841 = c^2$$

$$29 = c$$

$$\sin \theta =$$

$$= -\frac{21}{29}$$

$$\cos \theta =$$

$$= \frac{20}{29}$$

$$\csc \theta =$$

$$= -\frac{29}{21}$$

$$\sec \theta =$$

$$= \frac{29}{20}$$

$$\cot \theta =$$

$$= \frac{20}{21}$$

4. $\csc \theta = \sqrt{10}$; $\cot \theta > 0$

Quad III

$$1^2 + b^2 = (\sqrt{10})^2$$

$$1 + b^2 = 10$$

$$b^2 = 9$$

$$b = 3$$



$$\sin \theta =$$

$$= -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\cos \theta =$$

$$= -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\tan \theta =$$

$$= \frac{3}{1} = 3$$

$$\csc \theta =$$

$$= \frac{\sqrt{10}}{3}$$

$$\cot \theta =$$

$$= \frac{1}{3}$$

Directions: Use reference angles and the trigonometric function values for angles in special right triangles to find each trigonometric value.

5. $\cos 330^\circ$

$$\cos 30^\circ = \boxed{\frac{\sqrt{3}}{2}}$$

6. $\cot 225^\circ$

$$\cot 45^\circ = \boxed{1}$$

7. $\sin (-120^\circ)$

$$\sin 60^\circ = \boxed{-\frac{\sqrt{3}}{2}}$$

8. $\sec 135^\circ$

$$\sec 45^\circ = \frac{2}{-\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

9. $\csc \frac{7\pi}{6}$

$$\csc \frac{\pi}{6} = -\frac{2}{1} = \boxed{-2}$$

10. $\tan \left(-\frac{5\pi}{4}\right)$

$$\tan \frac{\pi}{4} = 1$$

11. $\cot \left(-\frac{3\pi}{2}\right)$

$$\cot \frac{\pi}{2} = \boxed{0}$$

Directions: Use the circle to find the value of each trigonometric function.

13. $\cos 300^\circ$

$$\frac{1}{2}$$

14. $\csc 120^\circ$

$$2$$

15. $\cot 270^\circ$

$$0$$

16. $\sin (-135^\circ)$

$$-\frac{\sqrt{2}}{2}$$

17. $\tan 90^\circ$

$$\text{undefined}$$

18. $\sec (-60^\circ)$

$$2$$

19. $\cos 210^\circ$

$$-\frac{1}{2}$$

20. $\sin \left(-\frac{11\pi}{6}\right)$

$$\frac{1}{2}$$

21. $\sec \frac{\pi}{6}$

$$\frac{2\sqrt{3}}{3}$$

22. $\tan \left(-\frac{2\pi}{3}\right)$

$$-\sqrt{3}$$

23. $\cos \left(-\frac{3\pi}{4}\right)$

$$-\frac{\sqrt{2}}{2}$$

24. $\csc \frac{5\pi}{3}$

$$-\frac{2\sqrt{3}}{3}$$

Name: _____

Pre-Calculus

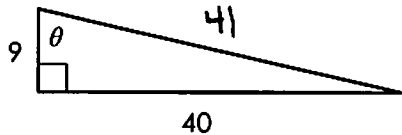
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Unit 5: Trigonometric Functions

Quiz 5-2: Trigonometric Functions

Give the exact values for the trigonometric function of angle θ .

1.



$$9^2 + 40^2 = c^2$$

$$1681 = c^2$$

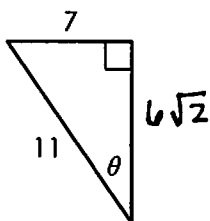
$$41 = c$$

$$\sin \theta = \frac{40}{41}$$

$$\cos \theta = \frac{9}{41}$$

$$\tan \theta = \frac{40}{9}$$

2.



$$11^2 = 7^2 + a^2$$

$$72 = a^2$$

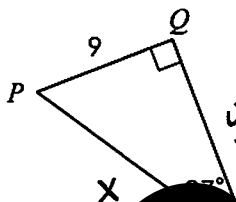
$$6\sqrt{2} = a$$

$$\sin \theta = \frac{6\sqrt{2}}{11}$$

$$\cos \theta = \frac{7}{11}$$

$$\tan \theta = \frac{6\sqrt{2}}{7}$$

3. Solve the triangle below. Round answers to the nearest tenth when necessary.



$$\sin 37 = \frac{9}{y}$$

$$y = \frac{9}{\sin 37}$$

$$y = 11.9$$

$$\tan 37 = \frac{9}{x}$$

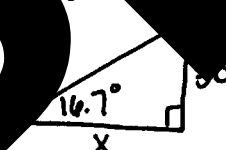
$$x = \frac{9}{\tan 37}$$

$$x = 11.9$$

$$QR = 11.9$$

$$PR = 15$$

$$m\angle P = 53^\circ$$

4. The angle of elevation from a ball on a football field to the top of a 16-foot goal post is $16^\circ 42'$. How far is the football from the base of the goal post? Round to the nearest tenth of a foot.

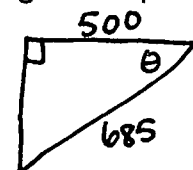
$$16^\circ 42' = 16.7$$

$$\tan 16.7 = \frac{16}{x}$$

$$x = \frac{16}{\tan 16.7} = 100$$

$$4. \underline{100 \text{ ft}}$$

5. A person is zip-lining in the rainforest. She is standing at the top of Platform A and looking down to zip-line to Platform B. If the horizontal distance between the platforms is 500 feet and the length of the zip-line is 685 feet, find the angle of depression from Platform A to Platform B to the nearest tenth.

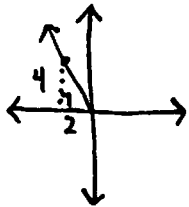


$$\cos \theta = \frac{500}{685}$$

$$\theta = \cos^{-1}\left(\frac{500}{685}\right) = 43.1^\circ$$

$$5. \underline{43.1^\circ}$$

6. $K(-2, 4)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ .



$$2^2 + 4^2 = r^2$$

$$20 = r^2$$

$$r = 2\sqrt{5}$$

$$x = -2$$

$$y = 4$$

$$r = 2\sqrt{5}$$

$$\sin \theta = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad \csc \theta = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

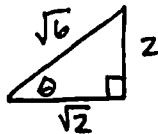
$$\cos \theta = \frac{-2}{2\sqrt{5}} = -\frac{\sqrt{5}}{5} \quad \sec \theta = \frac{2\sqrt{5}}{-2} = -\sqrt{5}$$

$$\tan \theta = \frac{4}{-2} = -2 \quad \cot \theta = \frac{-2}{4} = -\frac{1}{2}$$

7. If $\sin \theta < 0$ and $\sec \theta < 0$ in which quadrant could the terminal side of θ lie? 7. Quadrant III

8. If $\csc \theta = \frac{\sqrt{6}}{2}$, and $\cot \theta < 0$, find the exact value of $\sec \theta$.

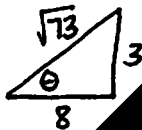
$$\sin \theta = \frac{2}{\sqrt{6}}$$



$$\sec \theta = \frac{\sqrt{6}}{2} = \sqrt{3}$$

8. $-\sqrt{3}$

9. $\tan \theta = -\frac{3}{8}$, and $\cos \theta > 0$, find the exact value of $\sin \theta$.



9. $-\frac{3\sqrt{13}}{73}$

Give the reference angle.

10. 285°

12. $-\frac{11\pi}{6}$

10. 75°

11. $\frac{4\pi}{9}$

12. $\frac{\pi}{6}$

Give the exact value of the trigonometric function.

13. $\sin 30^\circ$

14. $\cot 180^\circ$

15. $\cos(-60^\circ)$

13. $-\frac{1}{2}$

14. undef

15. $\frac{1}{2}$

16. $-\frac{\sqrt{3}}{2}$

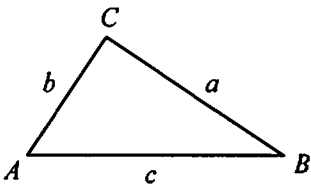
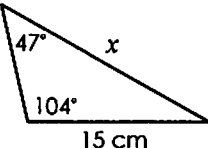
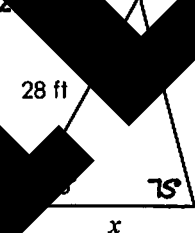
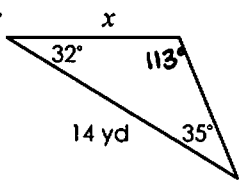
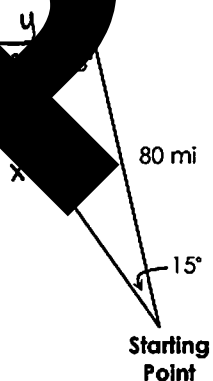
17. $\sqrt{2}$

18. $-\frac{\sqrt{3}}{3}$

16. $\frac{2}{3}$

17. $\sec\left(-\frac{7\pi}{4}\right)$

18. $\tan\frac{5\pi}{6}$

Name:		Date:	
Topic:		Class:	
Main Ideas/Questions		Notes/Examples	
LAW OF SINES 		Previously, you learned how to solve a right triangle. We now look at how to solve oblique (non-right) triangles. The Law of Sines can be used to find side and angle measures given two angles and any side (AAS or ASA), two side lengths and an angle outside of them (SSA). $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	
FINDING SIDES (Given AAS or ASA)		Directions: Find each missing measure. Round answers to the nearest tenth. 1.  $\frac{\sin 104}{x} = \frac{\sin 47}{15}$ $x \cdot \sin 47 = 15 \cdot \sin 104$ $x = 19.9 \text{ cm}$ 2.  $\frac{\sin 75}{28} = \frac{\sin 42}{x}$ $x \cdot \sin 75 = 28 \cdot \sin 42$ $x = 19.4 \text{ ft}$ 3.  $\frac{\sin 92}{11} = \frac{\sin 65}{x}$ $x \cdot \sin 92 = 11 \cdot \sin 65$ $x = 10 \text{ m}$ 4.  $\frac{\sin 113}{14} = \frac{\sin 35}{x}$ $x \cdot \sin 113 = 14 \cdot \sin 35$ $x = 8.7 \text{ yd}$	
		5. A pilot begins a flight traveling 80 miles northwest. To avoid a storm, he makes a turn, then returns to his original location. A diagram of his flight course is shown to the left. Find the total distance of the flight. $\frac{\sin 57}{80} = \frac{\sin 108}{x}$ $\frac{x \cdot \sin 57}{\sin 57} = \frac{80 \cdot \sin 108}{\sin 57}$ $x = 90.1$ $\frac{\sin 57}{80} = \frac{\sin 15}{y}$ $\frac{y \cdot \sin 57}{\sin 57} = \frac{80 \cdot \sin 15}{\sin 57}$ $y = 24.7$ Total: 195.4 mi	

FINDING ANGLES

(Given SSA)

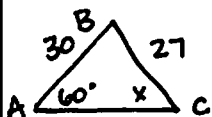
Watch out! When finding angles, it's possible to have one solution, two solutions, or no solution! This is called the **ambiguous case**.

Important Facts:

- The sine function always has a range between -1 and 1.
- Angles with a positive sine function value can lie in quad I or II.
- To find the other possible angle, subtract from 180°.
- The sum of the angle measures in a triangle is always 180°.
- No triangle can have two obtuse angles!

Directions: Information about $\triangle ABC$ is given below. Check your answers and give all possible solutions.

6. $m\angle A = 60^\circ$, $AB = 30$ in, $BC = 27$ in, find $m\angle C$



$$60 + 74.2 < 180 \checkmark$$

$$60 + 105.8 < 180 \checkmark$$

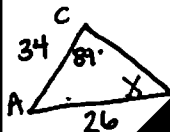
$$\frac{\sin 60}{27} = \frac{\sin x}{30}$$

$$30 \cdot \sin 60 = 27 \cdot \sin x$$

$$.9623 = \sin x$$

$$x = 74.2^\circ, 105.8^\circ$$

7. $m\angle C = 89^\circ$, $AC = 34$ in, $AB = 26$ in, find $m\angle B$



$$\frac{\sin 89}{26} = \frac{\sin x}{34}$$

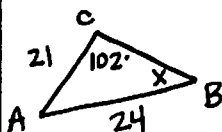
$$34 \cdot \sin 89 = 26 \cdot \sin x$$

$$33.75 = 26 \sin x$$

$$\sin x = 1.3075$$

No Solution

8. $m\angle C = 102^\circ$, $AC = 21$ yd, $AB = 24$ yd, find $m\angle B$



$$102 + 9.9 < 180 \checkmark$$

$$102 + 121.1 > 180 \times$$

$$\frac{\sin 102}{24} = \frac{\sin x}{21}$$

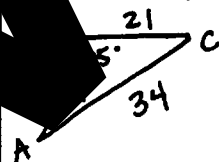
$$24 \cdot \sin 102 = 21 \cdot \sin x$$

$$21.8 = 21 \sin x$$

$$\sin x = 1.038$$

$$x = 111.2^\circ, 121.1^\circ$$

9. $m\angle B = 95^\circ$, $AC = 34$ km, $BC = 21$ km, find $m\angle A$



$$95 + 38 < 180 \checkmark$$

$$95 + 142 > 180 \times$$

$$\frac{\sin x}{21} = \frac{\sin 95}{34}$$

$$34 \cdot \sin x = 21 \cdot \sin 95$$

$$\sin x = .6153$$

$$x = 38^\circ, 142^\circ$$

10. $m\angle A = 30^\circ$, $AC = 29$ cm, $AB = 28$ cm, find $m\angle B$



$$30 + 31.2 < 180 \checkmark$$

$$30 + 148.8 < 180 \checkmark$$

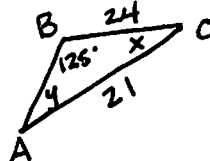
$$\frac{\sin 30}{28} = \frac{\sin x}{29}$$

$$28 \cdot \sin x = 29 \cdot \sin 30$$

$$\sin x = .5179$$

$$x = 31.2^\circ, 148.8^\circ$$

11. $m\angle B = 125^\circ$, $BC = 24$ ft, $AC = 21$ ft, find $m\angle C$



$$125 + 69.4 > 180 \times$$

$$125 + 110.6 > 180 \times$$

$$\frac{\sin 125}{21} = \frac{\sin y}{24}$$

$$24 \cdot \sin 125 = 21 \cdot \sin y$$

$$.9362 = \sin y$$

$$y = 69.4^\circ, 110.6^\circ$$

$m\angle C =$ Not possible

Name: _____

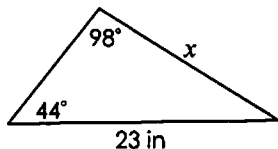
Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 7: Law of Sines

**** This is a 2-page document! ******Directions:** Find each missing measure. Round answers to the nearest tenth.

1.

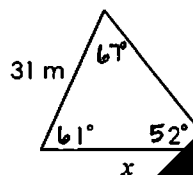


$$\frac{\sin 98}{23} = \frac{\sin 44}{x}$$

$$\frac{x \cdot \sin 98}{\sin 98} = \frac{23 \cdot \sin 44}{\sin 98}$$

$$x = 16.1 \text{ in}$$

2.

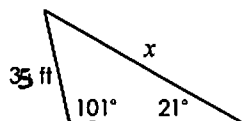


$$\frac{\sin 52}{31} = \frac{\sin 61}{x}$$

$$\frac{x \cdot \sin 52}{\sin 52} = \frac{31 \cdot \sin 61}{\sin 52}$$

$$x = 36.2 \text{ m}$$

3.



$$\frac{\sin 101}{x} = \frac{\sin 21}{35}$$

$$\frac{x \cdot \sin 21}{\sin 21} = \frac{35 \cdot \sin 101}{\sin 21}$$

$$x = 95.9 \text{ ft}$$

4.



$$\frac{39}{12} = \frac{\sin 19}{x}$$

$$\frac{x \cdot \sin 39}{\sin 39} = \frac{12 \cdot \sin 19}{\sin 39}$$

$$x = 6.2 \text{ km}$$

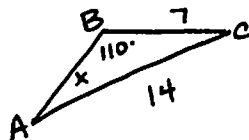
Directions: Information about $\triangle ABC$ is given below. Check your answers and give all possibilities.5. $m\angle A = 40^\circ$, $AB = 33$, $BC = 31$, find $m\angle C$ 

$$\frac{\sin 40}{31} = \frac{\sin x}{33}$$

$$33 \cdot \sin 40 = 31 \cdot \sin x$$

$$\sin x = .6843$$

$$x = 43.2^\circ, 136.8^\circ$$

6. $m\angle B = 110^\circ$, $AC = 14$ ft, $BC = 7$ ft, find $m\angle A$ 

$$28 + 110 < 180 \checkmark$$

$$152 + 110 > 180 \times$$

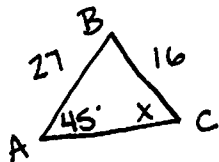
$$\frac{\sin 110}{14} = \frac{\sin x}{7}$$

$$7 \cdot \sin 110 = 14 \cdot \sin x$$

$$.4698 = \sin x$$

$$x = 28^\circ, 152^\circ$$

7. $m\angle A = 45^\circ$, $AB = 27$ mi, $BC = 16$ mi, find $m\angle C$



$$\frac{\sin x}{27} = \frac{\sin 45}{16}$$

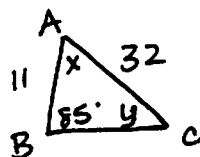
$$16 \cdot \sin x = 27 \cdot \sin 45$$

$$\sin x = 1.1932$$

$$x = \emptyset$$

No Solution

8. $m\angle B = 85^\circ$, $AC = 32$ in, $AB = 11$ in, find $m\angle A$



$$20 + 85 < 180 \checkmark$$

$$160 + 85 > 180 \checkmark$$

$$\frac{\sin y}{11} = \frac{\sin 85}{32}$$

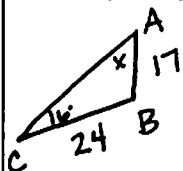
$$32 \cdot \sin y = 11 \cdot \sin 85$$

$$\sin y = 0.341$$

$$y = 19.9^\circ, 160.1^\circ$$

$$m\angle A = 15^\circ$$

9. $m\angle C = 16^\circ$, $CB = 24$ yd, $BA = 17$ yd, find $m\angle A$



$$16 + 22.9 < 180 \checkmark$$

$$16 + 167.1 < 180 \checkmark$$

$$\frac{\sin x}{24} = \frac{\sin 16}{17}$$

$$17 \cdot \sin x = 24 \cdot \sin 16$$

$$\sin x = .3891$$

$$x = 22.9^\circ, 157.1^\circ$$

10. $m\angle B = 144^\circ$, $AC = 4$ m, $BC = 9$ m, find $m\angle A$



$$\frac{\sin x}{4} = \frac{\sin 144}{9}$$

$$\sin x = 9 \cdot \sin 144$$

$$\sin x = 1.3225$$

$$x = \emptyset$$

No Solution

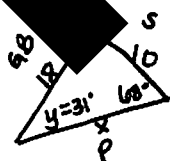
11. A rocket is launched from the Space Shuttle of Florida and can be seen from local schools. The angles of elevation from Vero Beach High School and Pine High School are $41^\circ 15'$ and $23^\circ 7' 30''$ respectively. If the two schools are 512 miles apart, how far is the rocket from Pine High School?

$$\frac{\sin 115.625}{512} = \frac{\sin 41.25}{x}$$

$$\frac{x \cdot \sin 115.625}{\sin 115.625} = \frac{512 \cdot \sin 41.25}{\sin 115.625}$$

$$x = 374.4 \text{ mi}$$

The lines of crops in a garden form a triangular shape. Strawberries are planted in a 10 ft line, green beans are planted in an 18 ft line. If the third side, a line of pumpkins, makes an 68° angle with the strawberries, determine the length of the line of pumpkins.



$$\frac{\sin 68}{18} = \frac{\sin y}{10}$$

$$10 \cdot \sin 68 = 18 \cdot \sin y$$

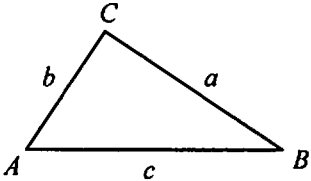
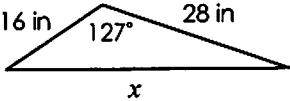
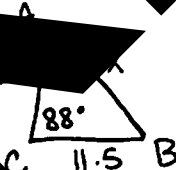
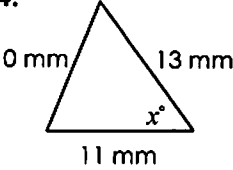
$$.5151 = \sin y$$

$$y = 31^\circ, 149^\circ$$

$$\frac{\sin 81}{x} = \frac{\sin 68}{18}$$

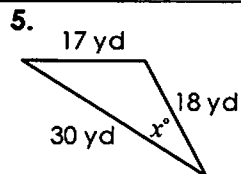
$$\frac{x \cdot \sin 68}{\sin 68} = \frac{18 \cdot \sin 81}{\sin 68}$$

$$x = 19.2 \text{ ft}$$

Name:	Date:
Topic:	Class:
Main Ideas/Questions	Notes/Examples
LAW OF COSINES 	The Law of Cosines can be used to find side and angle measures given two sides and the included angle (SAS) or three sides (SSS). $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ $b^2 = a^2 + c^2 - 2ac \cdot \cos B$ $c^2 = a^2 + b^2 - 2ab \cdot \cos C$
FINDING SIDES	Directions: Find each side length. Round answers to the nearest tenth. 1.  $x^2 = 16^2 + 28^2 - 2(16)(28) \cdot \cos 127$ $x^2 = 1040 - 912 \cos 127$ $x^2 = 179.226$ $x = 13.4$ 2.  $x^2 = 12^2 + 25^2 - 2(12)(25) \cdot \cos 36$ $x^2 = 769 - 600 \cos 36$ $x^2 = 283.5898$ $x = 16.8 \text{ mi}$ 3. In $\triangle ABC$, $\angle C = 88^\circ$, $AC = 22$ ft, and $BC = 11.5$ ft. Find AB.  $x^2 = 22^2 + 11.5^2 - 2(22)(11.5) \cos 88$ $x^2 = 616.25 - 506 \cos 88$ $x^2 = 598.5909$ $x = 24.5 \text{ ft}$
FINDING ANGLES	Directions: Find each angle measure. Round answers to the nearest tenth. 4.  $10^2 = 11^2 + 13^2 - 2(11)(13) \cos x$ $100 = 290 - 286 \cos x$ $-190 = -286 \cos x$ $.6643 = \cos x$ $x = 48.4^\circ$

APPLICATIONS

Note: A bearing measures the acute angle that a path or line of sight makes with a fixed north-south line.



$$17^2 = 30^2 + 18^2 - 2(30)(18) \cdot \cos x$$

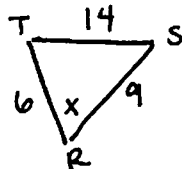
$$289 = 1224 - 1080 \cos x$$

$$-935 = -1080 \cos x$$

$$.8657 = \cos x$$

$$x = 30^\circ$$

6. In $\triangle RST$, $RS = 9$ cm, $ST = 14$ cm, and $RT = 6$ ft. Find $m\angle R$.



$$14^2 = 6^2 + 9^2 - 2(6)(9) \cdot \cos x$$

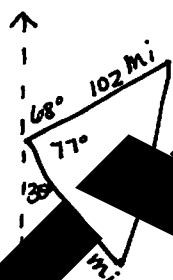
$$196 = 117 - 108 \cos x$$

$$79 = -108 \cos x$$

$$-.7315 = \cos x$$

$$x = 131^\circ$$

7. Two ships leave a port at 8 AM. Ship A leaves with a bearing of $N 68^\circ E$ at a speed of 17 mph. Ship B leaves with a bearing of $S 77^\circ E$ at a speed of 12 mph. If the ships maintain their paths, how far apart are they at 2 PM?



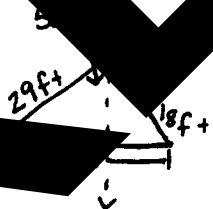
$$x^2 = 102^2 + 72^2 - 2(102)(72) \cdot \cos 135^\circ$$

$$x^2 = 15564 - 14832 \cos 135^\circ$$

$$x^2 = 12282.89$$

$$x = 110.8 \text{ mi}$$

8. Bruno and Aliona are figure skaters. Starting from the same location, Bruno skates 29 feet at a bearing of $S 55^\circ 9' W$. Aliona skates 18 feet at a bearing of $S 16^\circ E$. How far is Bruno now from Aliona?



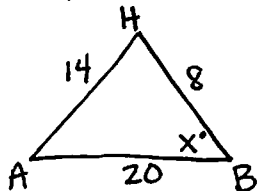
$$x^2 = 29^2 + 18^2 - 2(29)(18) \cdot \cos 72^\circ$$

$$x^2 = 1165 - 1044 \cos 72^\circ$$

$$x^2 = 842.3863$$

$$x = 29 \text{ ft}$$

9. A helicopter spots two landing pads below. The straight-line distances from the helicopter to pads is 14 miles to Landing Pad A and 8 miles to Landing Pad B. If the landing pads are 20 miles apart, find the angle of depression from the helicopter to Landing Pad B.



$$14^2 = 8^2 + 20^2 - 2(8)(20) \cos x$$

$$196 = 464 - 320 \cos x$$

$$-268 = -320 \cos x$$

$$.8375 = \cos x$$

$$x = 33.1^\circ$$

Name: _____

Unit 5: Trigonometric Functions

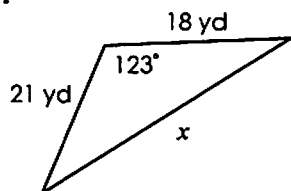
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Homework 8: Law of Cosines

** This is a 2-page document! **

Directions: Find the side length or angle measure. Round answers to the nearest tenth.

1.



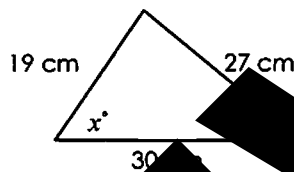
$$x^2 = 21^2 + 18^2 - 2(21)(18) \cos 123$$

$$x^2 = 765 - 756 \cos 123$$

$$x^2 = 1176.7471$$

$$x = 34.3 \text{ yd}$$

2.



$$27^2 = 19^2 + 30^2 - 2(19)(30) \cos x$$

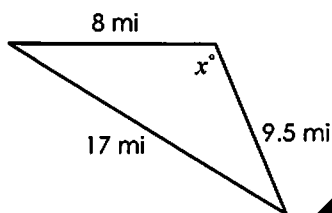
$$729 = 361 - 1140 \cos x$$

$$-542 = -1140 \cos x$$

$$.47 = \cos x$$

$$x = 62.2^\circ$$

3.



$$17^2 = 8^2 + 9.5^2 - 2(8)(9.5) \cos x$$

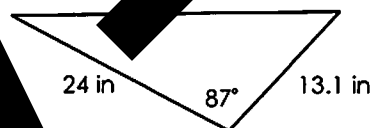
$$289 = 154.25 - 152 \cos x$$

$$134.75 = -152 \cos x$$

$$-.8865 = \cos x$$

$$x = 152.2^\circ$$

4.



$$x^2 = 24^2 + 13.1^2 - 2(24)(13.1) \cos 87$$

$$x^2 = 747.61 - 628.8 \cos 87$$

$$x^2 = 714.7012$$

$$x = 26.7 \text{ in}$$

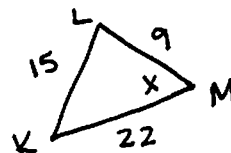
5. In $\triangle XYZ$, $\angle Z = 128^\circ$, $XZ = 26$ in, and $XY = 20$ in.

$$x^2 = 20^2 + 26^2 - 2(20)(26) \cos 128$$

$$x^2 = 1076 - 1040 \cos 128$$

$$x^2 = 1716.2879$$

$$x = 41.4 \text{ in}$$

6. In $\triangle KLM$, $KL = 15$ m, $KM = 22$ m, and $LM = 9$ m. Find $m\angle M$.

$$15^2 = 22^2 + 9^2 - 2(22)(9) \cos x$$

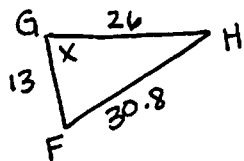
$$225 = 565 - 396 \cos x$$

$$-340 = -396 \cos x$$

$$.8586 = \cos x$$

$$x = 30.8^\circ$$

7. In $\triangle FGH$, $GH = 26$ mi, $FH = 30.8$ mi, and $FG = 13$ mi. Find $m\angle G$.



$$30.8^2 = 13^2 + 26^2 - 2(13)(26)\cos x$$

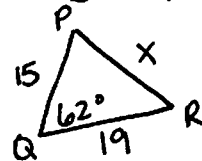
$$948.64 = 845 - 676\cos x$$

$$103.64 = -676\cos x$$

$$-.1533 = \cos x$$

$$x = 98.8^\circ$$

8. In $\triangle PQR$, $m\angle Q = 62^\circ$, $PQ = 15$ yd, and $RQ = 19$ yd. Find PR .



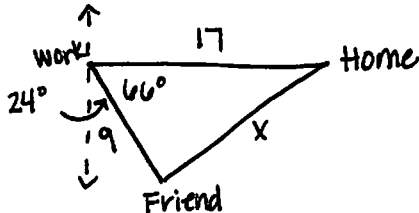
$$x^2 = 15^2 + 19^2 - 2(15)(19)\cos 62^\circ$$

$$x^2 = 586 - 570\cos 62^\circ$$

$$x^2 = 318.4012$$

$$x = 17.8$$

9. Michael leaves home and drives 17 miles west to work. From work, he travels 9 miles to a friend's house. If the bearing from work to the friend's house is $S 24^\circ E$, find the distance between the friend's house and home.



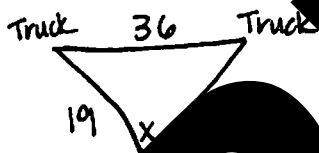
$$x^2 = 17^2 + 9^2 - 2(17)(9)\cos 66^\circ$$

$$x^2 = 306 - 306\cos 66^\circ$$

$$x^2 = 24.1771$$

$$x = 15.7$$

10. Two trucks, 36 feet apart, are towing a large car. If the length of one tow rope is 19 feet and the length of the other is 27 feet, find the angle formed by the two tow ropes.



$$36^2 = 19^2 + 27^2 - 2(19)(27)\cos x$$

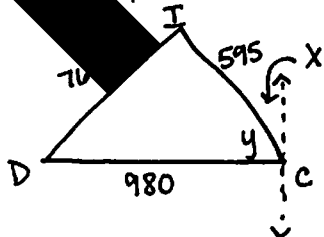
$$1296 = 1090 - 1026\cos x$$

$$206 = -1026\cos x$$

$$-.2008 = \cos x$$

$$x = 101.6^\circ$$

The cities of Dallas, Charleston, and Indianapolis form a triangle. The distance from Dallas to Charleston is 980 miles, Charleston to Indianapolis is 595 miles, and Indianapolis to Dallas is 764 miles. If Dallas and Charleston lie on the same latitude, find the bearing from Charleston to Indianapolis.



$$764^2 = 980^2 + 595^2 - 2(980)(595)\cos y$$

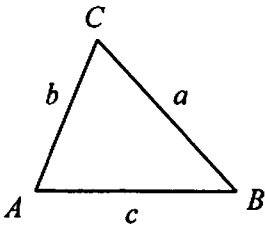
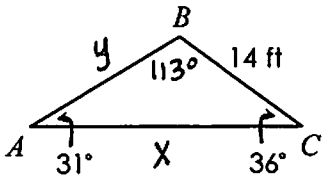
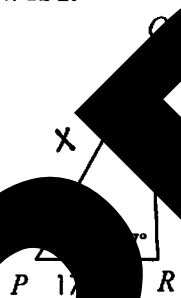
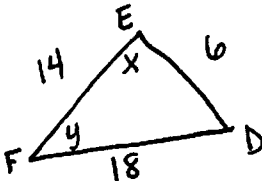
$$583696 = 1314425 - 1166200\cos y$$

$$-730729 = -1166200\cos y$$

$$.6266 = \cos y$$

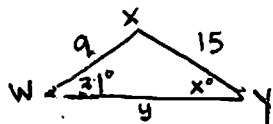
$$y = 51.2^\circ$$

$$N 38.8^\circ W$$

Name: _____		Date: _____	
Topic: _____		Class: _____	
Main Ideas/Questions		Notes/Examples	
LAW OF SINES & LAW OF COSINES 	LAW OF SINES $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ Use when given: <u>2 angles, 1 side (AAS)</u> <u>2 sides, 1 angle (SSA)</u>		LAW OF COSINES $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$ Use when given: <u>3 sides (SSS)</u> <u>2 sides, 1 angle (SAS)</u>
	EXAMPLE 1: 		Directions: Use the law of sines and/or law of cosines to solve each triangle. $\frac{\sin 31}{14} = \frac{\sin 113}{y}$ $y \sin 31 = 14 \sin 113$ $\frac{y \sin 31}{\sin 31} = \frac{14 \sin 113}{\sin 31}$ $y = 16$ $\frac{\sin 113}{14} = \frac{\sin 31}{x}$ $\sin 31 = \frac{14 \sin 113}{x}$ $x = 25$ $AB = 16 \text{ ft}$ $AC = 25 \text{ ft}$ $m\angle B = 113^\circ$
EXAMPLE 2: 	$x^2 = 28^2 + 32^2 - 2(28)(32) \cos 87$ $x^2 = 1024 - 1811.2 \cos 87$ $x^2 = 1024 - 162$ $x = 32$ $\frac{\sin 87}{32} = \frac{\sin y}{28}$ $28 \sin 87 = 32 \sin y$ $.8738 = \sin y$ $y = 60.9^\circ, 119.1^\circ$ $PQ = 32 \text{ km}$ $m\angle P = 60.9^\circ$ $m\angle Q = 32.1^\circ$		
EXAMPLE 3: In $\triangle DEF$, $DE = 14$ in, $DF = 6$ in, and $EF = 18$ in. 	$18^2 = 14^2 + 6^2 - 2(14)(6) \cos x$ $324 = 232 - 168 \cos x$ $92 = -168 \cos x$ $-.5476 = \cos x$ $x = 123.2^\circ$ $\frac{\sin 123.2}{18} = \frac{\sin y}{6}$ $18 \sin y = 6 \sin 123.2$ $\sin y = .2789$ $y = 16.2^\circ$ $m\angle D = 40.6^\circ$ $m\angle E = 123.2^\circ$ $m\angle F = 16.2^\circ$		

EXAMPLE 4:

In $\triangle WXY$:
 $WX = 9$ m, $XY = 15$ m,
 and $m\angle W = 31^\circ$



$$\frac{\sin 31}{15} = \frac{\sin x}{9}$$

$$15 \sin x = 9 \sin 31$$

$$\sin x = .3090$$

$$x = 18^\circ; 162^\circ$$

$$WY = 22 \text{ m}$$

$$m\angle X = 131^\circ$$

$$m\angle Y = 29^\circ$$

$$\begin{aligned} y^2 &= 9^2 + 15^2 - 2(9)(15) \cos 131 \\ y^2 &= 306 - 270 \cos 131 \\ y^2 &= 483.1359 \\ y &= 22 \end{aligned}$$

APPLICATIONS

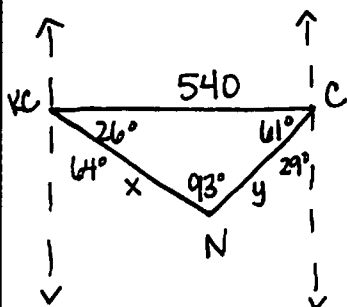
5. Cincinnati, Ohio is 540 miles due east of Kansas City, Missouri. The bearing from Kansas City to Nashville, Tennessee is $S 61^\circ W$. The bearing from Cincinnati to Nashville is $S 29^\circ W$.

a) Find the distance from Kansas City to Nashville.

$$\frac{\sin 93}{540} = \frac{\sin 61}{x}$$

$$\frac{x \sin 93}{\sin 93} = \frac{540 \sin 61}{\sin 93}$$

$$x = 472.9 \text{ mi}$$



b) Find the distance from Cincinnati to Nashville.

$$\frac{\sin 93}{540} = \frac{\sin 26}{y}$$

$$\frac{y \sin 93}{\sin 93} = \frac{540 \sin 26}{\sin 93}$$

$$y = 237 \text{ mi}$$

6. From the same port, Ship A sails 8 miles at a bearing of $N 45^\circ W$. Ship B sails 14 miles at a bearing of $S 70^\circ W$.

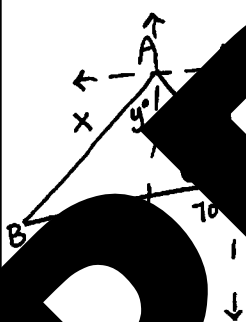
a) To the nearest tenth of a mile, how far is Ship A from Ship B?

$$x^2 = 14^2 + 8^2 - 2(14)(8) \cos 65$$

$$x^2 = 260 - 224 \cos 65$$

$$x^2 = 126.3335$$

$$x = 12.9 \text{ mi}$$



b) If Ship A sails to Ship B, what bearing should it take?

$$\frac{\sin w}{14} = \frac{\sin 65}{12.9}$$

$$\frac{12.9 \sin w}{12.9} = \frac{14 \sin 65}{12.9}$$

$$\sin w = .9836$$

$$w = 79.6^\circ$$

$$\text{Bearing} = S 34.6^\circ W$$

Name:

Date:

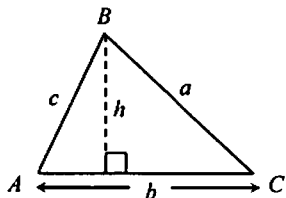
Topic:

Class:

Main Ideas/Questions

Notes/Examples

AREA OF TRIANGLES

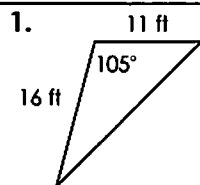


- What is the formula for the area of a triangle? $A = \frac{1}{2}bh$
- We know that $\sin C = \frac{h}{a}$
- Therefore, $h = a \cdot \sin C$

Substitute this into the area formula

$$A = \frac{1}{2}ab \cdot \sin C$$

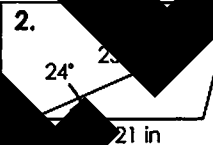
EXAMPLES

Directions: Find the area of each triangle rounded to the nearest tenth.

$$A = \frac{1}{2}(11)(16) \sin 105$$

$$A = 88 \sin 105$$

$$A = 85.12$$

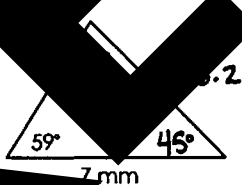


$$A = \frac{1}{2}(25)(21) \sin 24$$

$$A = 262.5 \sin 24$$

$$A = 106.8 \text{ in}^2$$

Note: You need to find the angle first using the Law of Sines for the last two examples.



$$\frac{\sin 59}{x} = \frac{\sin 76}{7}$$

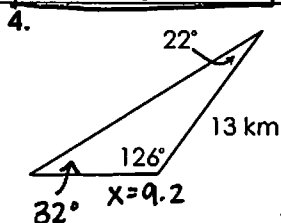
$$\frac{x \sin 76}{\sin 76} = \frac{7 \sin 59}{\sin 76}$$

$$x = 6.2$$

$$A = \frac{1}{2}(7)(6.2) \sin 45$$

$$A = 21.7 \sin 45$$

$$A = 15.3 \text{ mm}^2$$



$$\frac{\sin 32}{13} = \frac{\sin 22}{x}$$

$$\frac{x \sin 32}{\sin 32} = \frac{13 \sin 22}{\sin 32}$$

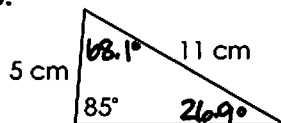
$$x = 9.2$$

$$A = \frac{1}{2}(9.2)(13) \sin 126$$

$$A = 59.8 \sin 126$$

$$A = 48.4 \text{ km}^2$$

5.



$$\frac{\sin x}{5} = \frac{\sin 85}{11}$$

$$\frac{5 \sin 85}{11} = \frac{11 \sin x}{11}$$

$$.4528 = \sin x$$

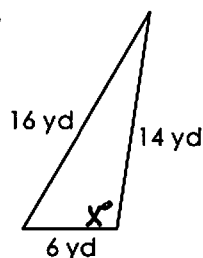
$$x = 27.0$$

$$A = \frac{1}{2}(5)(11) \sin(68.1)$$

$$A = 27.5 \sin(68.1)$$

$$A = 25.5 \text{ cm}^2$$

6.



$$16^2 = 6^2 + 14^2 - 2(6)(14) \cos x$$

$$256 = 232 - 168 \cos x$$

$$24 = -168 \cos x$$

$$-.1429 = \cos x$$

$$x = 98.2$$

$$A = \frac{1}{2}(6)(14) \sin 98.2$$

$$A = 41.6 \text{ yd}^2$$

HERON'S FORMULA

When the measures of all three sides of a triangle are given, you can use Heron's Formula to find the area:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$ semiperimeter

EXAMPLES

$$s = \frac{1}{2}(18 + 19 + 21) = 29$$

$$A = \sqrt{29(29-18)(29-19)(29-21)}$$

$$A = \sqrt{29(11)(10)(8)}$$

$$A = \sqrt{25520}$$

$$A = 159.7 \text{ mi}^2$$

8. A triangular-shaped piece of glass has side lengths that measure 35 inches, 27 inches, and 14 inches. Find the area of the glass to the nearest tenth of a square inch.

$$s = \frac{1}{2}(35 + 27 + 14)$$

$$s = 38$$

$$A = \sqrt{38(38-35)(38-27)(38-14)}$$

$$A = \sqrt{38(3)(11)(24)}$$

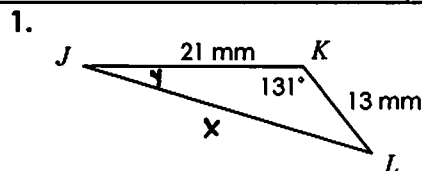
$$A = \sqrt{30096}$$

$$A = 173.5 \text{ in}^2$$

Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 9: Law of Sines & Law of Cosines;
Area of Triangles**** This is a 2-page document! ******Directions:** Use the Law of Sines and/or the Law of Cosines to solve each triangle and to nearest tenth when necessary.

$$x^2 = 21^2 + 13^2 - 2(21)(13) \cos 131$$

$$x^2 = 610 - 546 \cos 131$$

$$x^2 = 968.2082$$

$$x = 31.1$$

$$\frac{\sin 131}{31.1} = \frac{\sin y}{13}$$

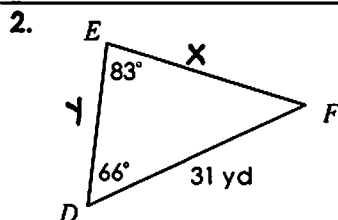
$$31.1 \sin y = 13 \sin 131$$

$$\sin y = 0.3155$$

$$y = 18.4^\circ, 161.6^\circ$$

$$m\angle J =$$

$$m\angle L = 30.6^\circ$$



$$\frac{\sin 83}{31} = \frac{\sin 66}{x}$$

$$\frac{x \sin 83}{\sin 83} = \frac{31 \sin 66}{\sin 83}$$

$$\frac{83}{31} = \frac{\sin y}{y}$$

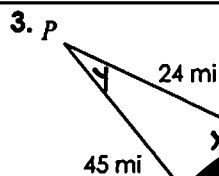
$$\frac{y \sin 83}{\sin 83} = \frac{31 \sin 31}{\sin 83}$$

$$y = 16.1$$

$$DE = 16.1 \text{ yd}$$

$$EF = 28.5 \text{ yd}$$

$$m\angle F = 31^\circ$$



$$x^2 = 24^2 + 45^2 - 2(24)(45) \cos 123.7$$

$$x^2 = 605 - 1296 \cos 123.7$$

$$x^2 = 1296.1$$

$$x = 36.0$$

$$\frac{\sin 123.7}{36} = \frac{\sin y}{24}$$

$$\sin y = 0.4992$$

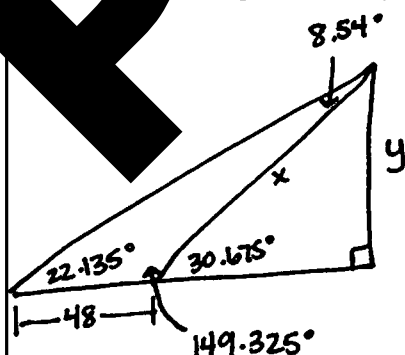
$$y = 29.9^\circ, 150.1^\circ$$

$$m\angle P = 29.9^\circ$$

$$m\angle Q = 123.7^\circ$$

$$m\angle R = 26.4^\circ$$

Jason and Allison are hiking in the woods when they spot a rare owl in a tree. Jason stops and measures an angle of elevation of $22^\circ 8' 6''$. At the same time, Allison is standing 48 feet closer to the tree, measuring an angle of elevation of $30^\circ 40' 30''$ to the owl. If Jason and Allison are the same height, their eyes 5 feet from the ground, find the height of owl in the tree.



$$\frac{\sin 8.54}{48} = \frac{\sin 22.135}{x}$$

$$x \sin 8.54 = 48 \sin 22.135$$

$$x = 121.8$$

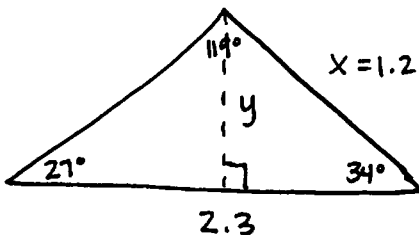
$$\frac{\sin 30.675}{y} = \frac{\sin 90}{121.8}$$

$$y \sin 90 = 121.8 \sin 30.675$$

$$y = 62.1$$

owl = 67.1 ft high

5. A drone is flying over a college campus. The drone spots the cafeteria on the west end of campus at a 27° angle of depression and the sports complex on the east end of campus at a 34° angle of depression. If the cafeteria and sports complex are separated by a straight road 2.3 mi long, how high is the drone?



$$\frac{\sin 119}{2.3} = \frac{\sin 27}{x}$$

$$\frac{x \sin 119}{\sin 119} = \frac{2.3 \sin 27}{\sin 119}$$

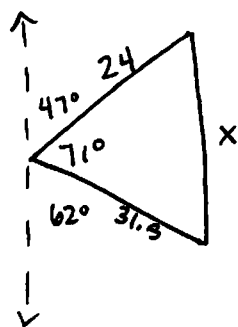
$$x = 1.2$$

$$\frac{\sin 90}{1.2} = \frac{\sin 34}{y}$$

$$\frac{y \sin 90}{\sin 90} = \frac{1.2 \sin 34}{\sin 90}$$

$$y = 0.67 \text{ mi}$$

6. Two bicyclists left at 11:30 AM for a ride. Rider A traveled at a constant rate of 12 mph at a bearing of $N 47^\circ E$, stopping at 1:30 PM. Rider B traveled at a constant rate of 9 mph at a bearing of $S 62^\circ E$. Determine the distance between the two bicyclists.



$$x^2 = 24^2 + 31.5^2 - 2(24)(31.5)\cos 71$$

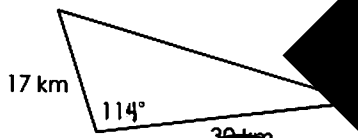
$$x^2 = 1568.25 - 1512 \cos 71$$

$$x^2 = 1075.99$$

$$x = 32.8 \text{ mi apart}$$

Directions: Find the area of each triangle. Round to the nearest tenth.

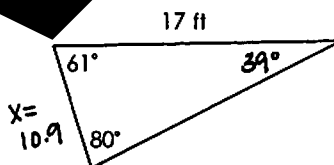
7.



$$A = \frac{1}{2}(17)(30)\sin 114$$

$$A = 233.3$$

$$A = 233$$



$$x = 10.9$$

$$\frac{\sin 80}{17} = \frac{\sin 39}{x}$$

$$\frac{x \sin 80}{\sin 80} = \frac{17 \sin 39}{\sin 80}$$

$$x = 10.9$$

$$A = \frac{1}{2}(10.9)(17)\sin 61$$

$$A = 92.65 \sin 61$$

$$A = 81.4$$

9. In $\triangle ABC$, $AB = 26$ m, $\angle A = 127^\circ$, and $m\angle B = 41^\circ$. Find its area.



$$\frac{\sin 12}{26} = \frac{\sin 41}{x}$$

$$\frac{x \sin 12}{\sin 12} = \frac{26 \sin 41}{\sin 12}$$

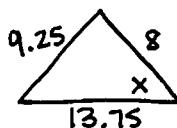
$$x = 82$$

$$A = \frac{1}{2}(26)(82)\sin 127$$

$$A = 1066 \sin 127$$

$$A = 851.3 \text{ mi}^2$$

10. If a triangular garden has sides of 8 ft, 9 ft 3 in, and 13 ft 9 in, use Heron's Formula to find its area.



$$s = \frac{1}{2}(9.25 + 8 + 13.75)$$

$$s = 15.5$$

$$A = \sqrt{15.5(15.5-9.25)(15.5-8)(15.5-13.75)}$$

$$A = \sqrt{1271.4848}$$

$$A = 35.7 \text{ ft}^2$$

Name: _____

Pre-Calculus

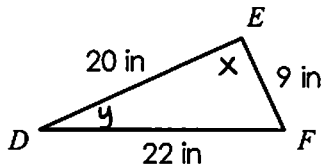
Date: _____ Per: _____

Unit 5: Trigonometric Functions

Quiz 5-3: Law of Sines & Law of Cosines

Solve each triangle using the Law of Sines and Law of Cosines. Round measures to the nearest tenth.

1.



$$22^2 = 20^2 + 9^2 - 2(20)(9) \cos x$$

$$3 = -360 \cos x$$

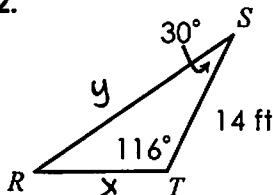
$$x = 90.5$$

$$\frac{\sin 90.5}{22} = \frac{\sin y}{9}$$

$$\sin y = .4091$$

$$y = 24.5, 155.4$$

2.



$$\frac{\sin 34}{14} = \frac{\sin 30}{x}$$

$$x \sin 34 = 14 \sin 30$$

$$x = 12.5$$

$$\frac{\sin 116}{14} = \frac{\sin 34}{y}$$

$$\sin 34 = \frac{14 \sin 116}{y}$$

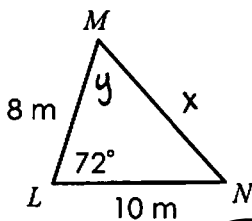
$$y = 27.5$$

$$m\angle S = 116^\circ$$

$$RT = 12.5 \text{ ft}$$

$$m\angle R = 34^\circ$$

3.



$$x^2 = 8^2 + 10^2 - 2(8)(10) \cos 72$$

$$x^2 = 55.73$$

$$x = 7.47$$

$$\frac{\sin x}{10} = \frac{\sin 72}{8}$$

$$\sin x = .8888$$

$$x = 62.7^\circ, 117.3^\circ$$

$$MN = 10.7 \text{ m}$$

$$m\angle M = 62.7^\circ$$

$$m\angle N = 45.3^\circ$$

Using the information in $\triangle ABC$, give all possible solutions for the angle measure.4. $m\angle C = 64^\circ$, $BC = 19$, $AB = 18$, find $m\angle B$

$$\frac{\sin x}{19} = \frac{\sin 64}{18}$$

$$\frac{18 \sin x}{18} = \frac{19 \sin 64}{18}$$

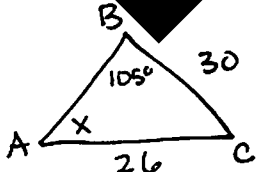
$$\sin x = .9487$$

$$x = 71.6^\circ, 108.4^\circ$$

$$4. m\angle B = 71.6^\circ, 108.4^\circ$$

$$64 + 71.6 < 180 \checkmark$$

$$64 + 108.4 < 180 \checkmark$$

5. $m\angle B = 105^\circ$, $BC = 30$ yd, $AC = 26$ yd, find $m\angle A$ 

$$\frac{\sin x}{30} = \frac{\sin 105}{26}$$

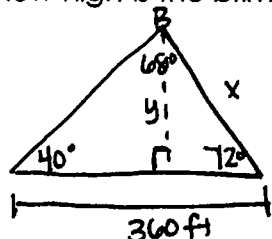
$$\frac{26 \sin x}{26} = \frac{30 \sin 105}{26}$$

$$\sin x = 1.145$$

$$x = \text{no solution}$$

$$5. m\angle A = \text{No Solution}$$

6. A blimp is flying directly above a football field. The angle of depression to the base of one goal post is 40° . The angle of depression to the base of the other goal post is 72° . If the goal posts are 360 feet apart, how high is the blimp flying? Round to the nearest tenth of a foot.



$$\frac{\sin 68^\circ}{360} = \frac{\sin 40^\circ}{x}$$

$$x \sin 68^\circ = 360 \sin 40^\circ$$

$$x = 249.6$$

$$\frac{\sin 90^\circ}{249.6} = \frac{\sin 72^\circ}{y}$$

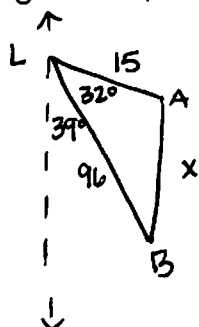
$$y \sin 90^\circ = 249.6 \sin 72^\circ$$

$$y = 237.4$$

6. 237.4 ft

7. 83.7 km

7. The bearing from a lighthouse to Ship A is $S 71^\circ E$. The bearing from the lighthouse to Ship B is $S 39^\circ E$. If Ship A is 15 kilometers from the lighthouse and Ship B is 9 kilometers from the lighthouse, find the distance between the ships.



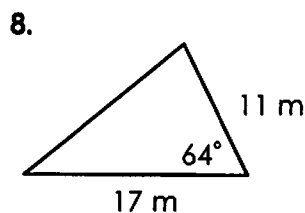
$$x^2 = 15^2 + 9^2 - 2(15)(9) \cos 32^\circ$$

$$x^2 = 9441 - 2880 \cos 32^\circ$$

$$x^2 = 6998.67$$

$$x = 83.7$$

Find the area of each triangle. Round answers to the nearest square unit.



$$A = \frac{1}{2} (11)(17) \sin 64^\circ$$

$$A = 84$$

8. 84 m²

9. 25.9 ft²

10. 68.1 km²

9.



$$A = \frac{1}{2} (12)(5.8) \sin 104^\circ$$

$$A = 34.8 \sin 104^\circ$$

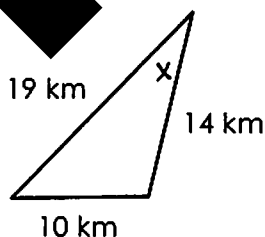
$$A = 25.9$$

$$A = \frac{1}{2} (12)(5.8) \sin 104^\circ$$

$$A = 34.8 \sin 104^\circ$$

$$A = 25.9$$

10.



$$10^2 = 14^2 + 19^2 - 2(14)(19) \cos X$$

$$100 = 557 - 532 \cos X$$

$$.8590 = \cos X$$

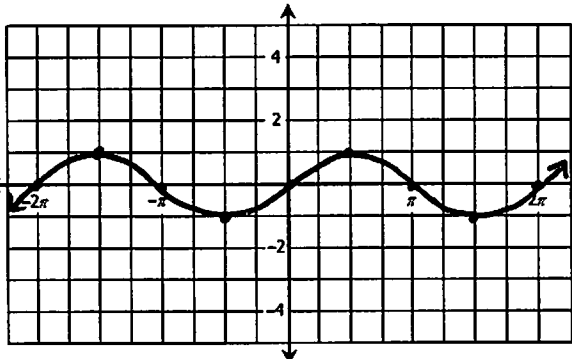
$$X = 30.8^\circ$$

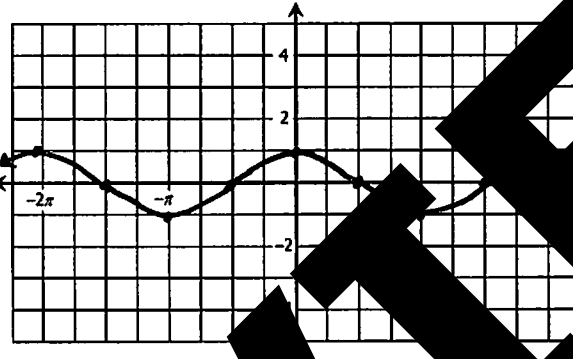
$$A = \frac{1}{2} (14)(19) \sin 30.8^\circ$$

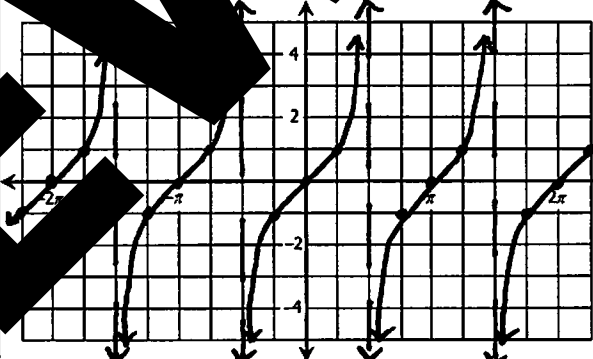
$$A = 133 \sin 30.8^\circ$$

$$A = 68.1$$

Trigonometric Functions

SINE	
Parent Function: $f(x) = \sin x$	
	
Domain: $(-\infty, \infty)$	
Range: $[-1, 1]$	
x-intercept(s): $n\pi, n \in \mathbb{Z}$	
y-intercept: 0	
Asymptotes: None	
Amplitude: 1	Period: 2π
Transformations: $f(x) = a \sin b(x-h) + k$	
Amplitude: $ a $	Period: $\frac{2\pi}{ b }$
Horizontal (Phase) Shift: $-h$ shifts right, h shifts left	Vertical Shift: $+k$ shifts up, $-k$ shifts down

COSINE	
Parent Function: $f(x) = \cos x$	
	
Domain: $(-\infty, \infty)$	
Range: $[-1, 1]$	
x-intercept(s): $\frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	
y-intercept: 1	
Asymptotes: None	
Amplitude: 1	Period: 2π
Transformations: $f(x) = a \cos b(x-h) + k$	
Amplitude: $ a $	Period: $\frac{2\pi}{ b }$
Horizontal (Phase) Shift: $-h$ shifts right, h shifts left	Vertical Shift: $+k$ shifts up, $-k$ shifts down

TANGENT	
Parent Function: $f(x) = \tan x$	
	
Domain: $x \in \mathbb{R}; x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	
Range: $(-\infty, \infty)$	
x-intercept(s): $n\pi, n \in \mathbb{Z}$	
y-intercept: 0	
Asymptotes: $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	
Amplitude: Undefined	Period: π
Transformations: $f(x) = a \tan b(x-h) + k$	
Amplitude: Undefined	Period: $\frac{\pi}{ b }$
Horizontal (Phase) Shift: $-h$ shifts right, h shifts left	Vertical Shift: $+k$ shifts up, $-k$ shifts down

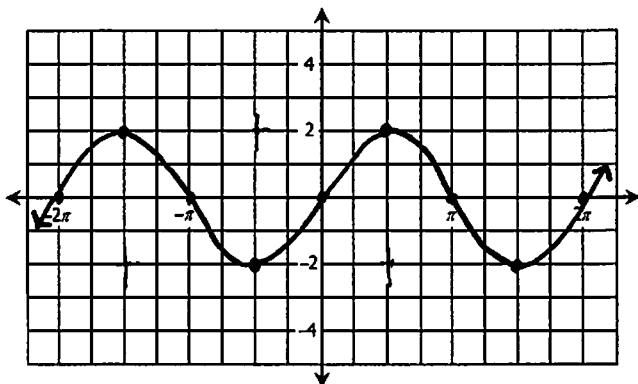
Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
Graphing SINE & COSINE Functions	SINE $f(x) = \sin x$	COSINE $f(x) = \cos x$
	Notice that the cosine function is a horizontal reflection of the sine function. Any transformation of a sine function is called a sinusoid. Therefore, sine and cosine functions are sinusoidal functions.	
PERIOD & AMPLITUDE	Functions that repeat at regular intervals are called periodic functions.	
	Period: the length of a cycle; can start at any point on the graph	
	Amplitude: half the distance between the minimum and maximum values	
	For $f(x) = a \cdot \sin bx$ or $f(x) = a \cdot \cos bx$, the amplitude is a and the period is $\frac{2\pi}{ b }$.	
	Identify the amplitude and period of each function.	
	1. $f(x) = \sin 4x$ amp: 1 per = $\frac{2\pi}{4} = \frac{\pi}{2}$	2. $f(x) = \frac{7}{2} \cdot \cos x$ amp = $\frac{7}{2}$ per = $\frac{2\pi}{1} = 2\pi$
	3. $f(x) = -5 \cdot \cos \frac{1}{3}x$ amp = 5 per = $\frac{2\pi}{\frac{1}{3}} = 6\pi$	4. $f(x) = -2 \cdot \sin \frac{4}{3}x$ amp = 2 per = $\frac{2\pi}{\frac{4}{3}} = \frac{3\pi}{2}$

Directions: Identify the amplitude and period, then graph the function.

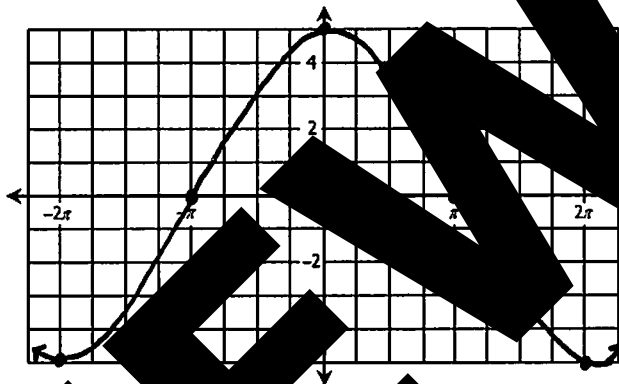
5. $f(x) = 2 \cdot \sin x$

Amplitude: 2 Period: 2π



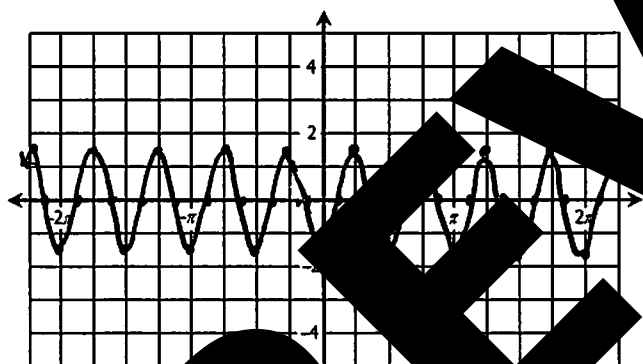
6. $f(x) = 5 \cdot \cos \frac{1}{2}x$

Amplitude: 5 Period: 4π



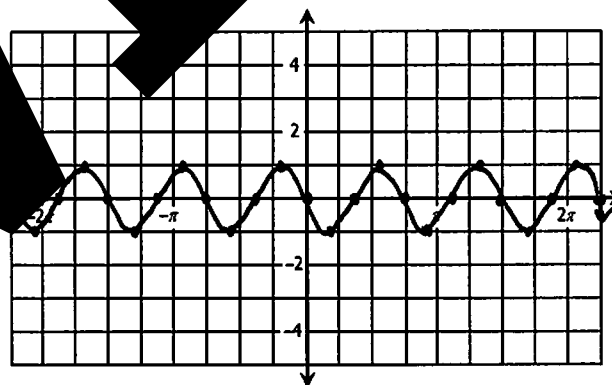
7. $f(x) = -\frac{3}{2} \cdot \cos 4x$

Amplitude: $\frac{3}{2}$ Period: $\frac{\pi}{2}$



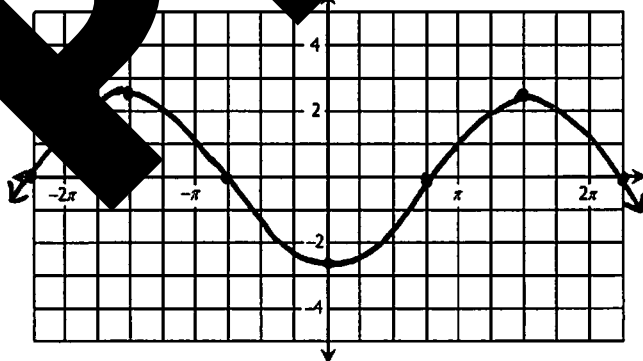
8. $f(x) = -\sin \frac{8}{3}x$

Amplitude: 1 Period: $\frac{3\pi}{4}$



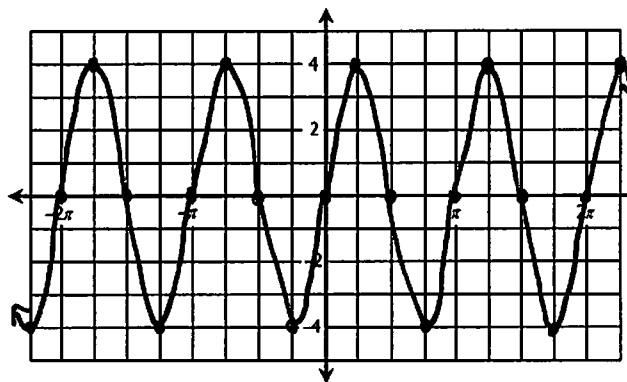
9. $f(x) = \cos \frac{2}{3}x$

Amplitude: 1 Period: 3π



10. $f(x) = 4 \cdot \sin 2x$

Amplitude: 4 Period: π

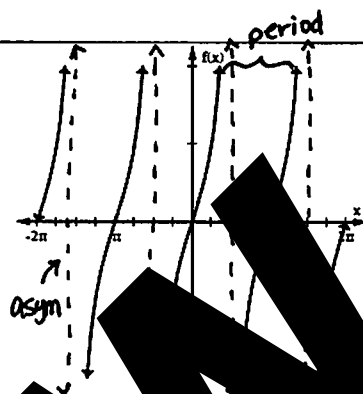


Graphing TANGENT Functions

TANGENT $f(x) = \tan x$

The tangent function is undefined for certain values of x . Therefore, the tangent function has **vertical asymptotes**.

The tangent function does not have an amplitude.

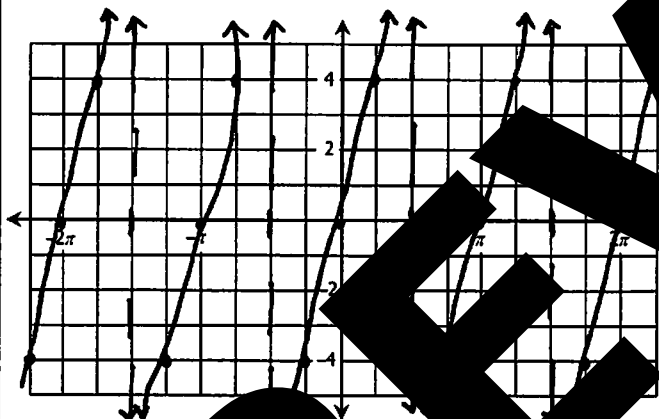


Given $f(x) = a \cdot \tan bx$, the period is _____ and the asymptotes are odd multiples of _____.

Directions: Identify the period, then graph each function. Show the asymptotes on the graph.

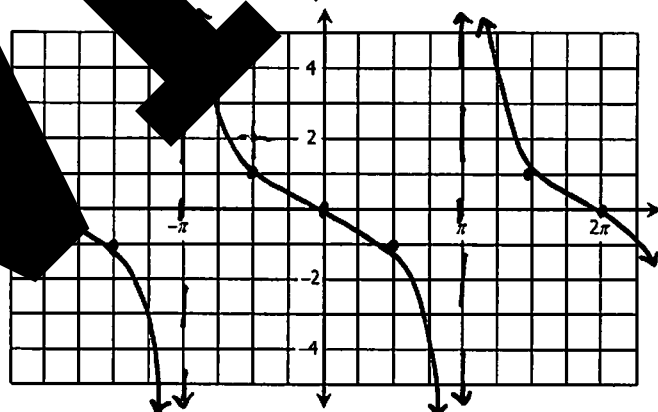
11. $f(x) = 4 \cdot \tan x$

Period: π



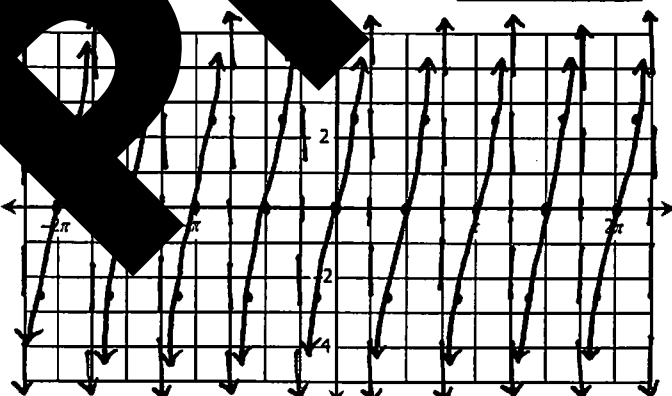
12. $f(x) = -\tan \frac{x}{2}$

Period: 2π



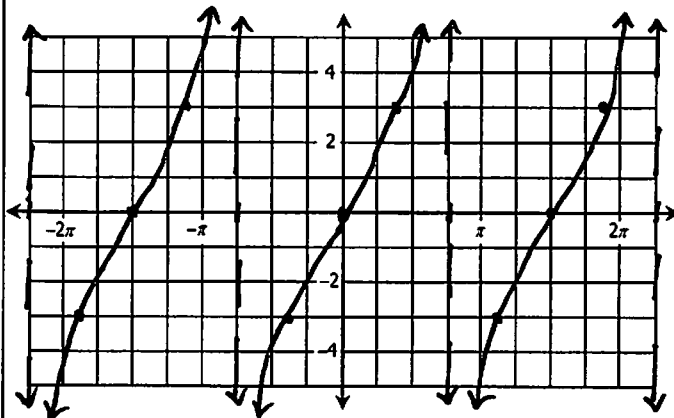
13. $f(x) = \tan 2x$

Period: $\frac{\pi}{2}$



14. $f(x) = 3 \cdot \tan \frac{2}{3}x$

Period: $\frac{3\pi}{2}$



Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

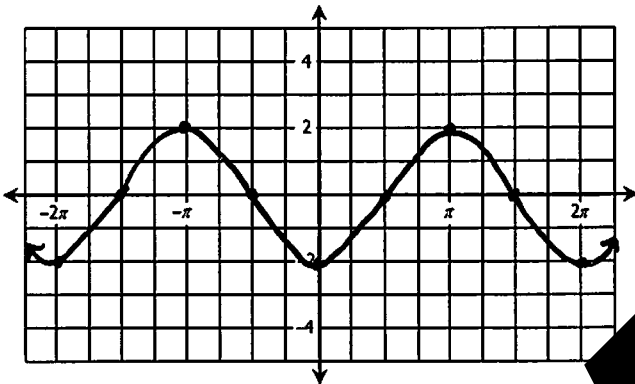
Homework 10: Graphing Sine, Cosine, & Tangent Functions

**** This is a 2-page document! ****

Directions: Identify the amplitude and period, then graph the function.

1. $f(x) = -2 \cdot \cos x$

Amplitude: 2 Period: 2π



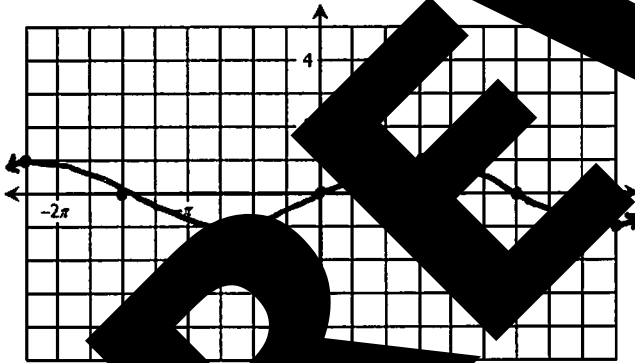
2. $f(x) = \frac{5}{2} \cdot \sin 2x$

Amplitude: $\frac{5}{2}$ Period: π



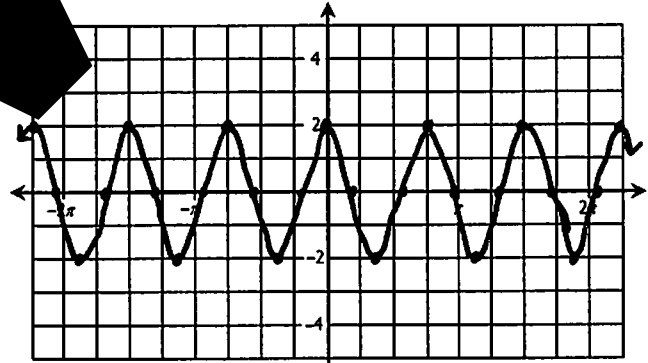
3. $f(x) = \sin \frac{2}{3}x$

Amplitude: 1 Period: 3π



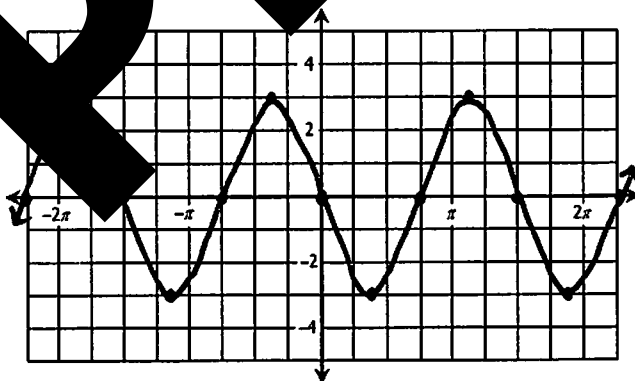
4. $f(x) = 2 \cdot \cos \frac{4}{3}x$

Amplitude: 2 Period: $\frac{3\pi}{2}$



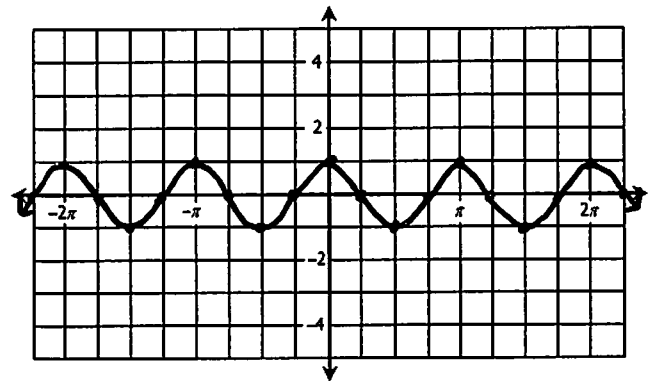
5. $f(x) = 3 \cdot \sin \frac{\pi}{3}x$

Amplitude: 3 Period: $\frac{3\pi}{2}$



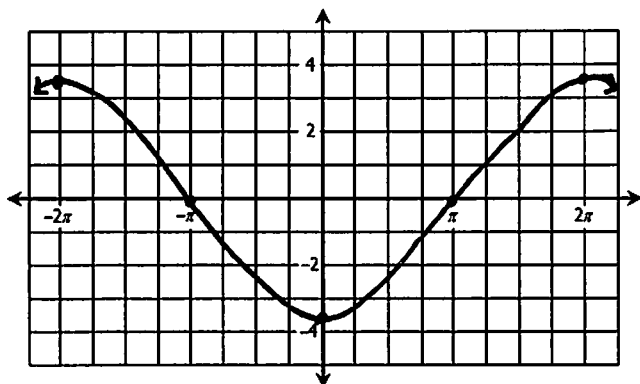
6. $f(x) = \cos 2x$

Amplitude: 1 Period: π



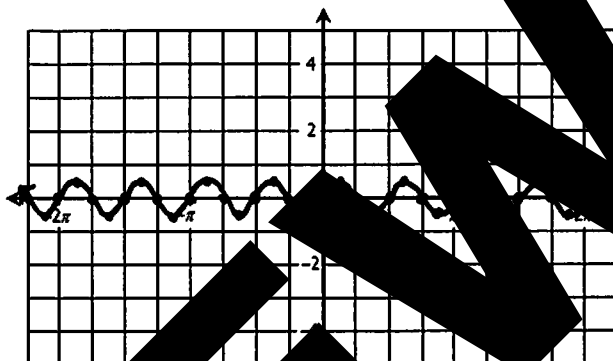
$$7. f(x) = -\frac{7}{2} \cdot \cos \frac{1}{2}x$$

Amplitude: $\frac{7}{2}$ Period: 4π



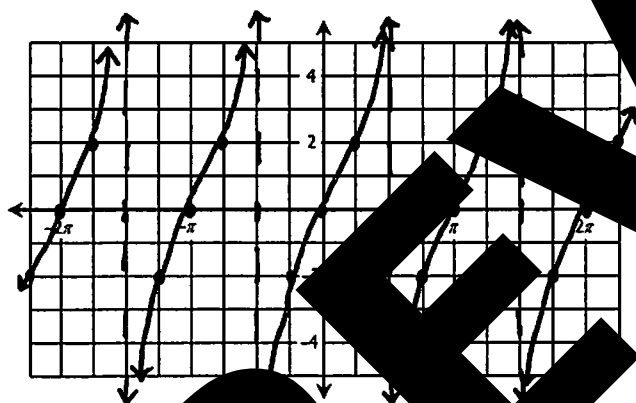
$$8. f(x) = \frac{1}{2} \cdot \sin 4x$$

Amplitude: $\frac{1}{2}$ Period: $\frac{\pi}{2}$



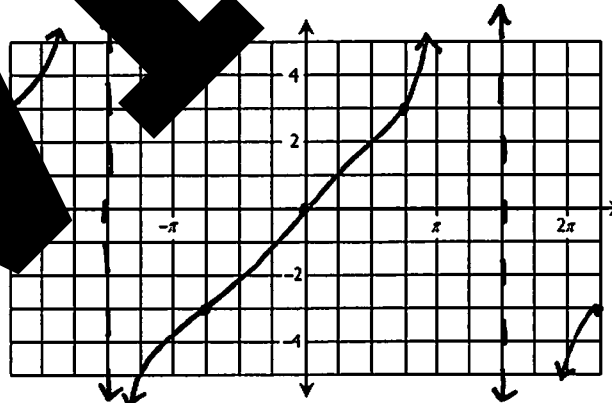
$$9. f(x) = 2 \tan x$$

Amplitude: undef Period: π



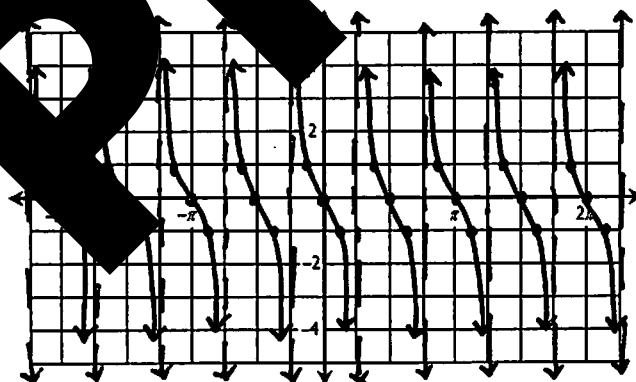
$$10. f(x) = 3 \tan \frac{x}{3}$$

Amplitude: undef Period: 3π



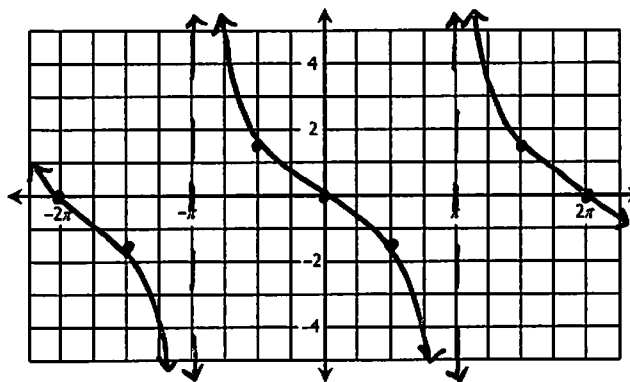
$$11. f(x) = \tan 12x$$

Amplitude: undef Period: $\frac{\pi}{12}$



$$12. f(x) = -\frac{3}{2} \tan \frac{1}{2}x$$

Amplitude: undef Period: 2π



Name:

Date:

Topic:

Class:

Main Ideas/Questions

Notes/Examples

TRANSLATIONS

of Trigonometric Functions

$$f(x) = a \cdot \sin b(x-h) + k$$

$$f(x) = a \cdot \cos b(x-h) + k$$

$$f(x) = a \cdot \tan b(x-h) + k$$

Like other functions, trigonometric functions can be translated.
Given the functions to the left, h represents a horizontal shift
and k represents a vertical shift.

Vertical Shifts

$+k$ shifts up
 $-k$ shifts down

Horizontal (Phase) Shifts

$+h$ shifts left
 $-h$ shifts right

Describe the the translations of each trigonometric function.

1. $f(x) = \cos x + 2$

up 2

2. $f(x) = \tan x$

down 1

3. $f(x) = \sin\left(x + \frac{\pi}{2}\right)$

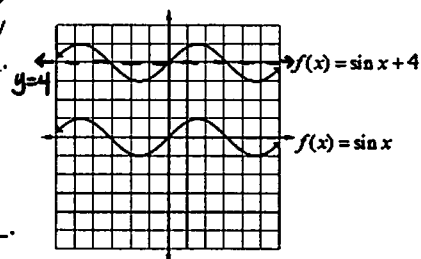
left $\frac{\pi}{2}$

4. $f(x) = \cos(x - \pi)$

Right π

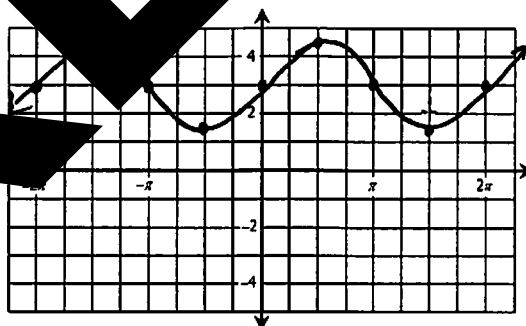
MIDLINE

- Shifting a graph vertically will create a new horizontal line called midline.
- The new reference line which a graph oscillates.

The equation for the midline is $y = k$.

5. $f(x) = \frac{3}{2} \cdot \sin x + 3$

Graph the function and identify each characteristic.

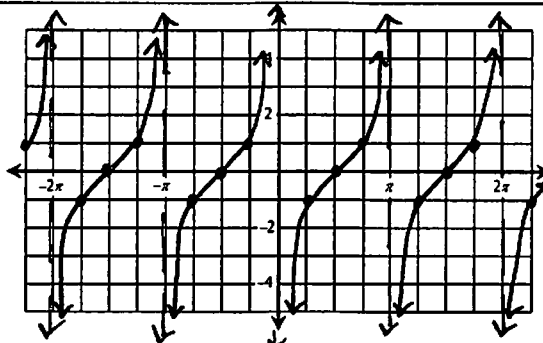
Amplitude: $\frac{3}{2}$ Period: 2π

Phase Shift: None

Vertical Shift: up 3

Midline: $y = 3$

$f(x) = \tan\left(x + \frac{\pi}{2}\right)$



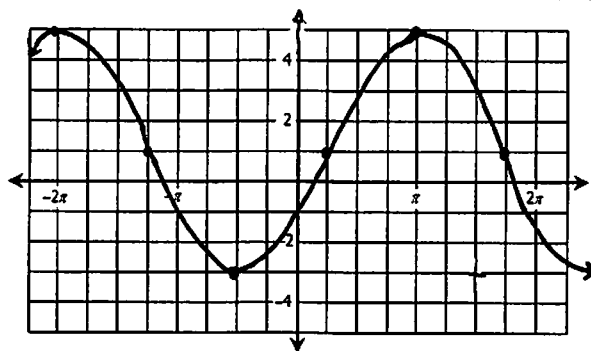
Amplitude: undef

Period: π Phase Shift: left $\frac{\pi}{2}$

Vertical Shift: None

Midline: $y = 0$

7. $f(x) = 4 \cdot \cos \frac{2}{3}(x - \pi) + 1$



Amplitude: 4

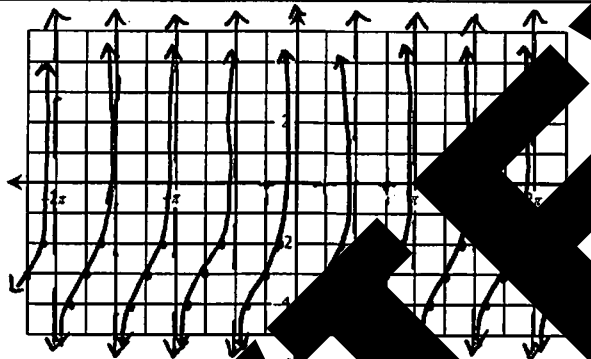
Period: 3π

Phase Shift: Right π

Vertical Shift: up 1

Midline: $y = 1$

8. $f(x) = \tan 2\left(x + \frac{\pi}{4}\right) - 3$



Amplitude: undef

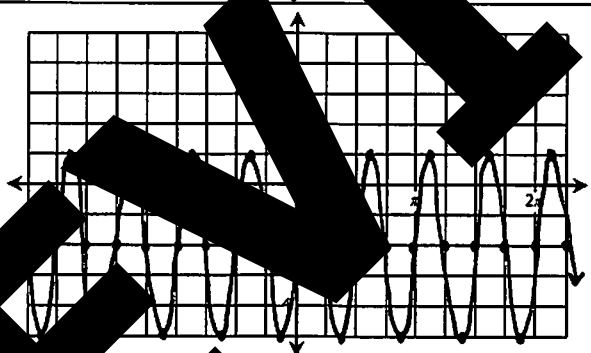
Period: $\frac{\pi}{2}$

Phase Shift: Left $\frac{\pi}{4}$

Vertical Shift: down 3

Midline: $y = -3$

9. $f(x) = 3 \cdot \sin 4\left(x - \frac{\pi}{2}\right) - 2$



Amplitude: 3

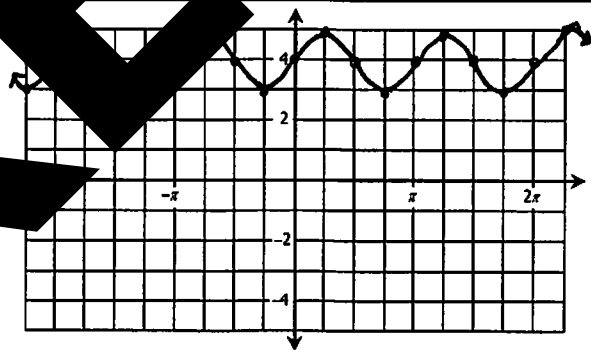
Period: $\frac{\pi}{2}$

Phase Shift: Right $\frac{\pi}{2}$

Vertical Shift: down 2

Midline: $y = -2$

10. $f(x) = \cos 2\left(x - \frac{3\pi}{4}\right) + 4$



Amplitude: 1

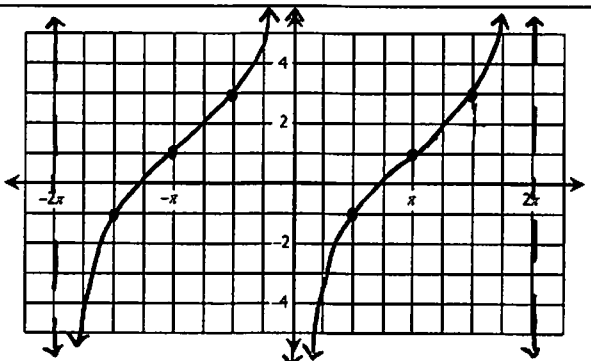
Period: π

Phase Shift: Left $\frac{3\pi}{4}$

Vertical Shift: up 4

Midline: $y = 4$

11. $f(x) = 2 \cdot \tan \frac{1}{2}(x + \pi) + 1$



Amplitude: undef

Period: 2π

Phase Shift: Left π

Vertical Shift: up 1

Midline: $y = 1$

Name: _____

Unit 5: Trigonometric Functions



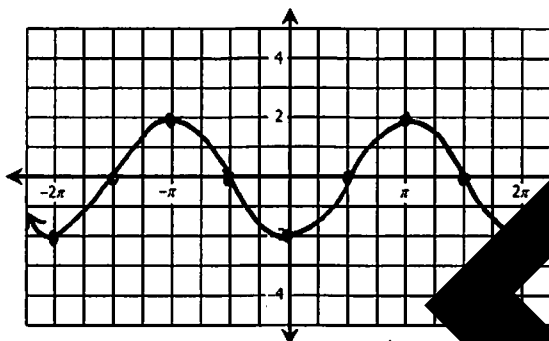
Date: _____ Per: _____

Homework 11: Translating Trigonometric Functions

** This is a 2-page document! **

Directions:

1. $f(x) = 2 \cdot \cos(x + \pi)$



Amplitude: 2

Period: 2π Phase Shift: $-\pi$

Vertical Shift: None

Midline: $y = 0$

Amplitude: 1

Period: 2π

Phase Shift: None

Vertical Shift: Up 4

Midline: $y = 4$

2. $f(x) = \sin x + 4$



Amplitude: undef

Period: $\frac{\pi}{2}$ Phase Shift: Right $\frac{3\pi}{4}$

Vertical Shift: None

Midline: $y = 0$

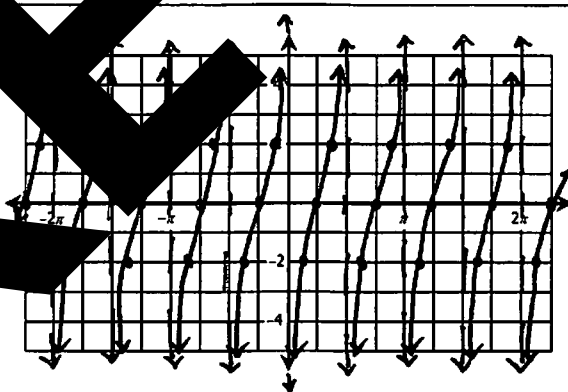
Amplitude: 1

Period: 4π Phase Shift: Right $\frac{\pi}{2}$

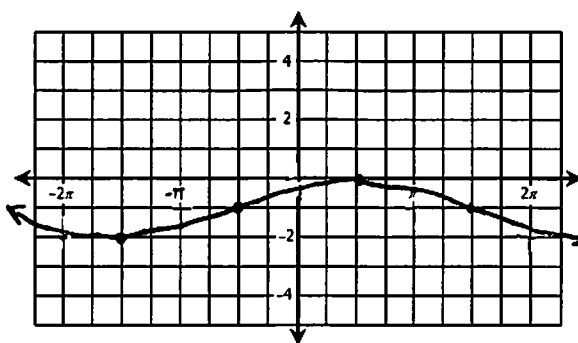
Vertical Shift: down 1

Midline: $y = -1$

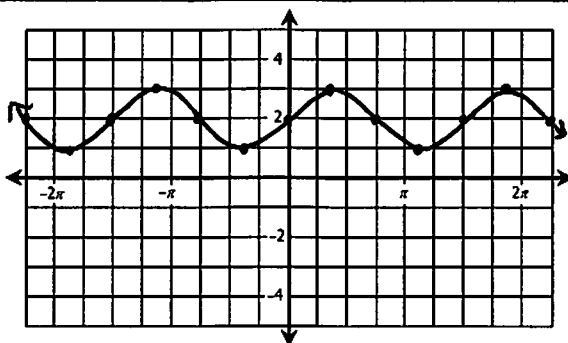
3. $f(x) = 2 \cdot \tan 2\left(x - \frac{3\pi}{4}\right)$



$$\frac{1}{2}\left(x - \frac{\pi}{2}\right) - 1$$



5. $f(x) = \sin\left(\frac{4}{3}\left(x + \frac{3\pi}{2}\right) + 2\right)$



Amplitude: 1

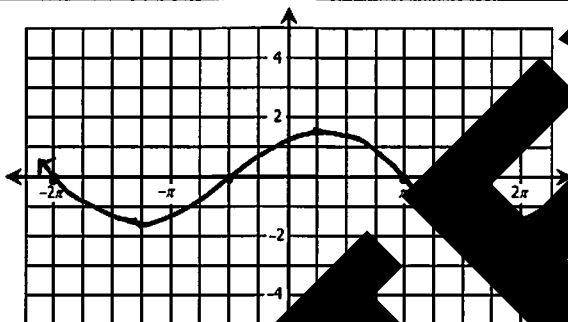
Period: $\frac{3\pi}{2}$

Phase Shift: $\frac{3\pi}{2}$

Vertical Shift: up

Midline: $y = 2$

6. $f(x) = \frac{3}{2} \cdot \cos\left(\frac{2}{3}\left(x - \frac{\pi}{4}\right)\right)$



Amplitude: $\frac{3}{2}$

Period: 3π

Phase Shift: Right $\frac{\pi}{4}$

Vertical Shift: None

Midline: $y = 0$

7. $f(x) = 3 \cdot \tan\left(\frac{1}{3}(x + \pi) + 1\right)$



Amplitude: undef

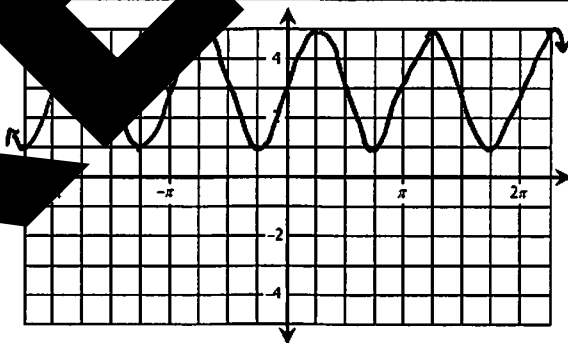
Period: 3π

Phase Shift: Left π

Vertical Shift: up 1

Midline: $y = 1$

8. $f(x) = -2 \cdot \cos\left(x + \frac{5\pi}{4}\right) + 3$



Amplitude: 2

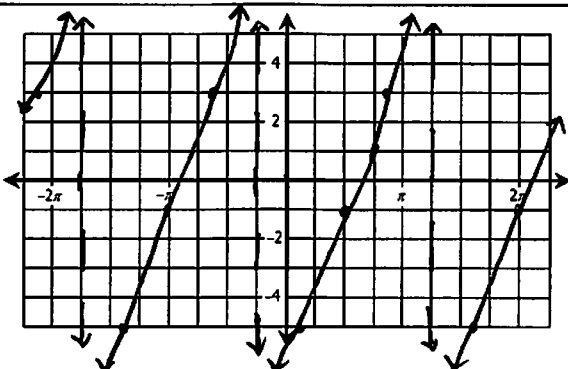
Period: π

Phase Shift: Left $\frac{5\pi}{4}$

Vertical Shift: up 3

Midline: $y = 3$

9. $f(x) = \tan\left(\frac{2}{3}\left(x - \frac{\pi}{2}\right) - 1\right)$



Amplitude: undef

Period: $\frac{3\pi}{2}$

Phase Shift: Right $\frac{\pi}{2}$

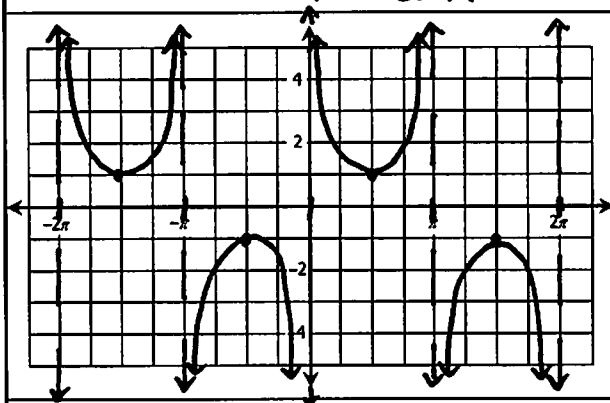
Vertical Shift: down 1

Midline: $y = 1$

Trigonometric Functions

COSECANT

Parent Function: $f(x) = \csc x$



Domain: $x \in \mathbb{R}; x \neq n\pi, n \in \mathbb{Z}$

Range: $(-\infty, -1]$ and $[1, \infty)$

x-intercept(s): None

y-intercept: None

Asymptotes: $x = n\pi, n \in \mathbb{Z}$

Amplitude: Undefined

Period: 2π

Transformations:

$$f(x) = a \csc b(x-h) + k$$

Amplitude:

undefined

Period:

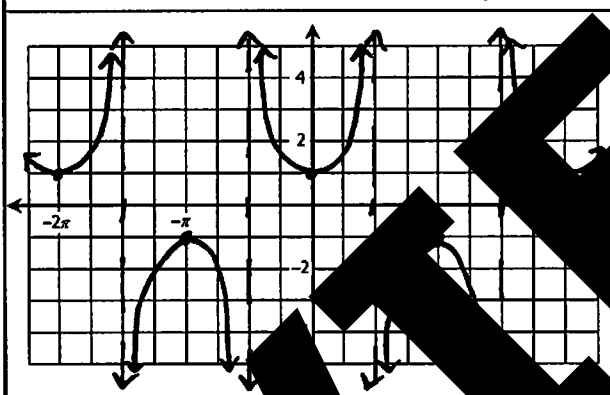
$$\frac{\pi}{|b|}$$

Horizontal (Phase) Shift:
-h shifts right
+h shifts left

Vertical Shift:
+k shifts up
-k shifts down

SECANT

Parent Function: $f(x) = \sec x$



Domain: $x \in \mathbb{R}; x \neq n\pi, n \in \mathbb{Z}$

Range: $(-\infty, -1]$ and $[1, \infty)$

x-intercept(s): None

y-intercept: None

Asymptotes: $x = n\pi, n \in \mathbb{Z}$

Amplitude: Undefined

Period: 2π

Transformations:

$$f(x) = a \sec b(x-h) + k$$

Amplitude:

undefined

Period:

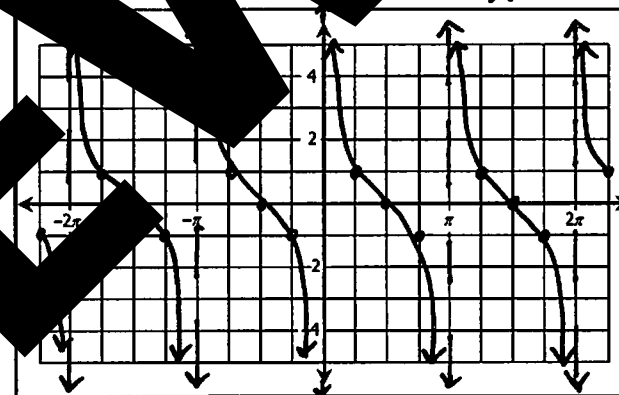
$$\frac{2\pi}{|b|}$$

Horizontal (Phase) Shift:
-h shifts right
+h shifts left

Vertical Shift:
+k shifts up
-k shifts down

COTANGENT

Parent Function: $f(x) = \cot x$



Domain: $x \in \mathbb{R}; x \neq n\pi, n \in \mathbb{Z}$

Range: $(-\infty, \infty)$

x-intercept(s): $\frac{\pi}{2} + n\pi, n \in \mathbb{Z}$

y-intercept: None

Asymptotes: $x = n\pi, n \in \mathbb{Z}$

Amplitude: Undefined

Period: π

Transformations:

$$f(x) = a \cot b(x-h) + k$$

Amplitude:

undefined

Period:

$$\frac{\pi}{|b|}$$

Horizontal (Phase) Shift:
-h shifts right
+h shifts left

Vertical Shift:
+k shifts up
-k shifts down

Name:

Date:

Topic:

Class:

Main Ideas/Questions

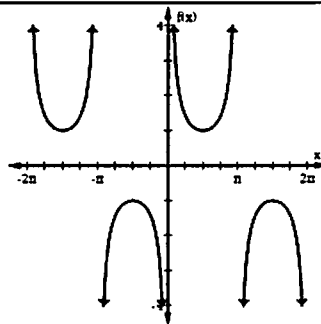
Notes/Examples

Graphing RECIPROCAL Functions

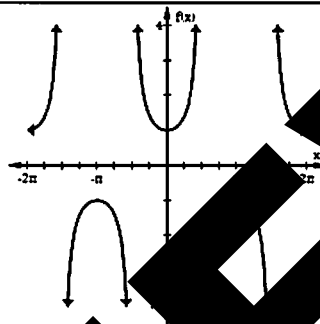
*All reciprocal functions have vertical asymptotes.

*Reciprocal functions do not have an amplitude.

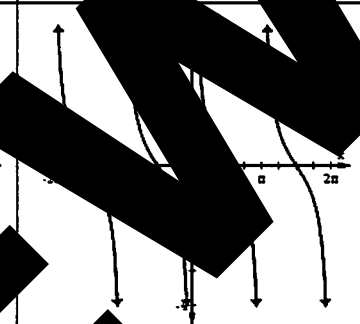
COSECANT $f(x) = \csc x$



SECANT $f(x) = \sec x$



COTANGENT $f(x) = \cot x$



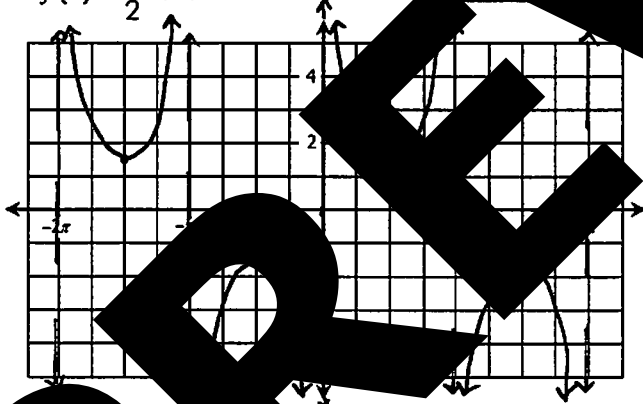
Given $f(x) = a \cdot \csc bx$, $f(x) = a \cdot \sec bx$, or $f(x) = a \cdot \cot bx$:

- The period of cosecant, secant, and cotangent functions is $\frac{2\pi}{|b|}$.
- The period of the tangent function is $\frac{\pi}{|b|}$.

Directions: Identify the period, then graph each function. Show the asymptotes on the graph.

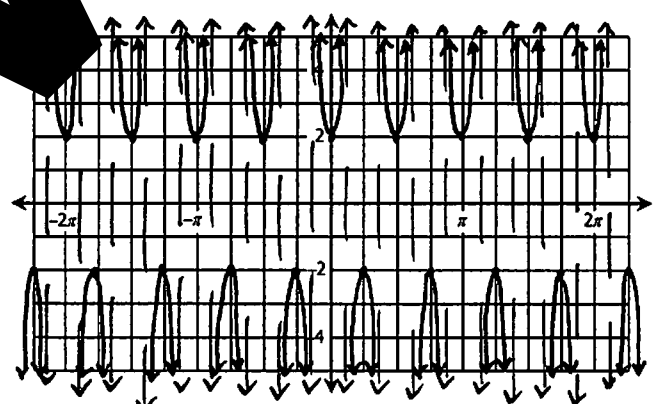
1. $f(x) = \frac{3}{2} \cdot \csc x$

Period: π



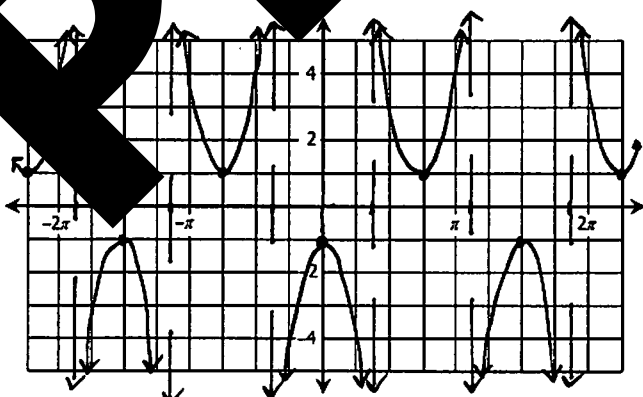
$f(x) = 2 \cdot \sec 4x$

Period: $\frac{\pi}{2}$



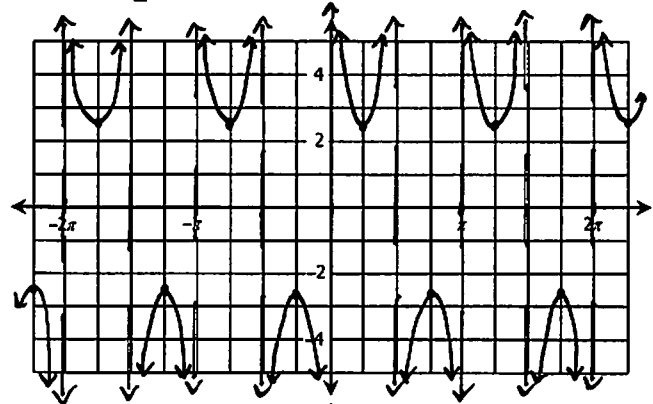
$f(x) = \sec \frac{4}{3}x$

Period: $\frac{3\pi}{2}$

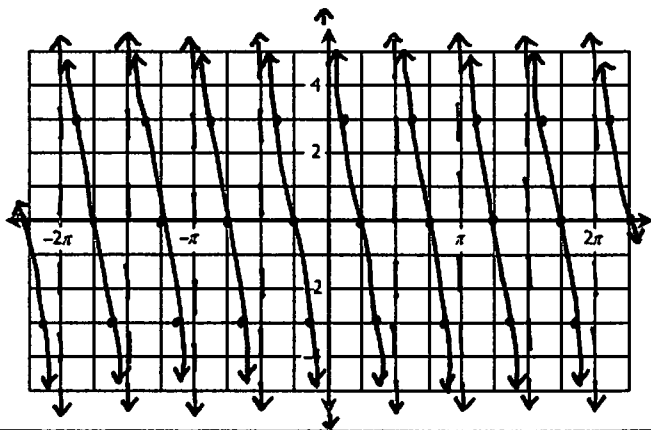


4. $f(x) = \frac{5}{2} \cdot \csc 2x$

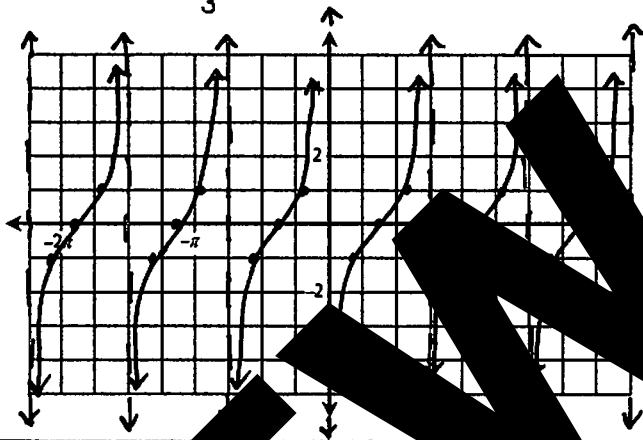
Period: π



5. $f(x) = 3 \cdot \cot 2x$ Period: $\frac{\pi}{2}$

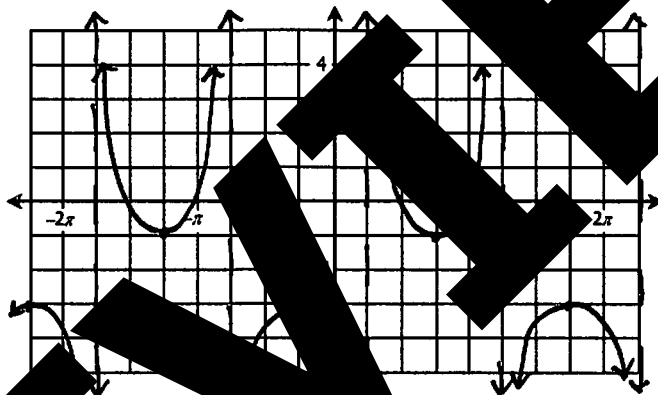


6. $f(x) = -\cot \frac{4}{3}x$ Period: $\frac{3\pi}{4}$



Directions: Graph each function and show the asymptotes on each graph. Identify the period, phase shift, vertical shift, and midline of each function.

7. $f(x) = \csc\left(x - \frac{\pi}{4}\right) - 2$



Period:

2π

Phase Shift:

Right $\frac{\pi}{4}$

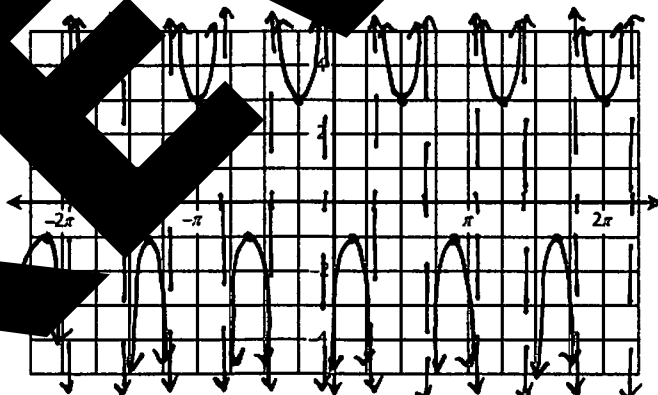
Vertical Shift:

down 2

Midline:

$y = -2$

8. $f(x) = 2 \cdot \sec \frac{8}{3}\left(x + \pi\right) + 1$



Period:

$\frac{3\pi}{4}$

Phase Shift:

Left π

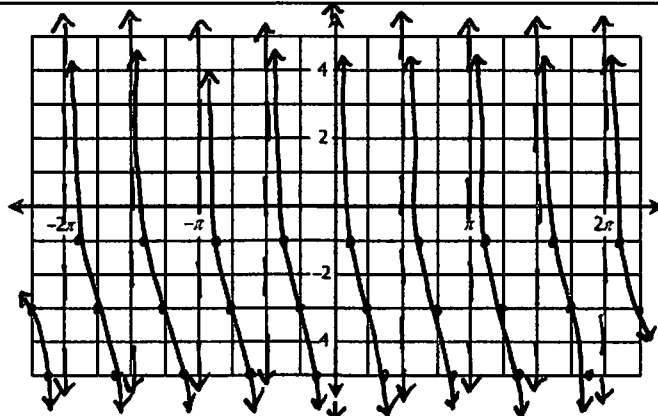
Vertical Shift:

up 1

Midline:

$y = 1$

$f(x) = 2\left(x - \frac{3\pi}{2}\right) - 3$



Period:

$\frac{\pi}{2}$

Phase Shift:

Right $\frac{3\pi}{2}$

Vertical Shift:

down 3

Midline:

$y = -3$

Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

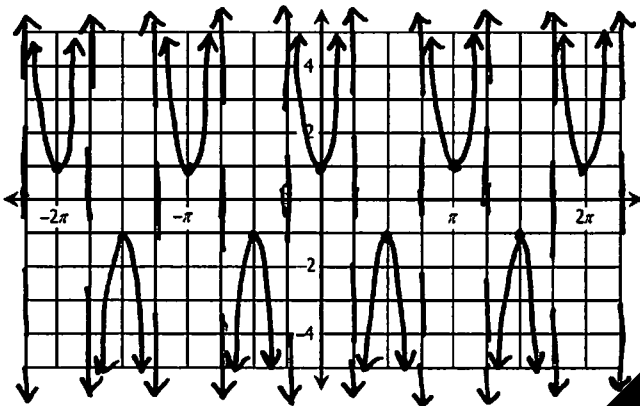
Homework 12: Graphing Reciprocal Functions

**** This is a 2-page document! ****

Directions:

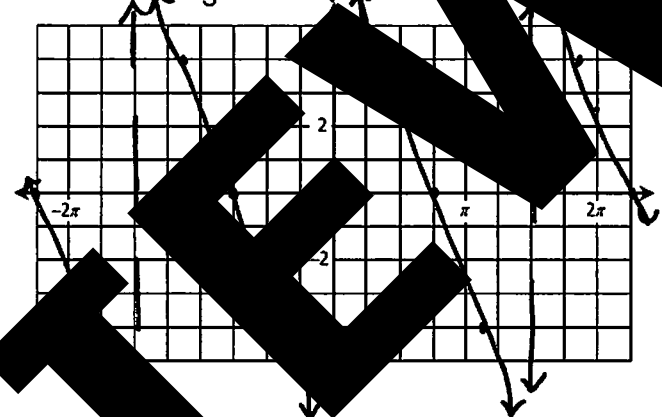
1. $f(x) = \sec 2x$

Period: π



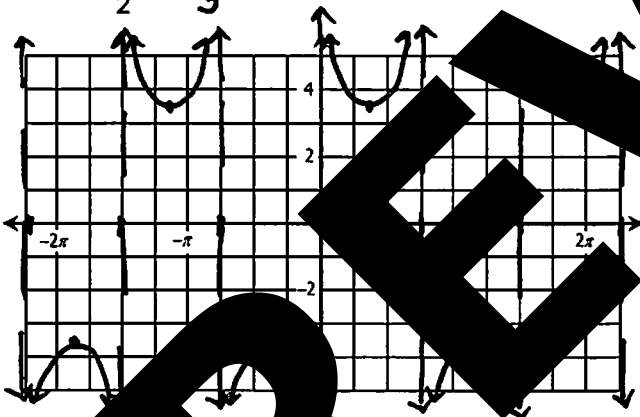
2. $f(x) = 4 \cdot \cot \frac{2}{3}x$

Period: 3π



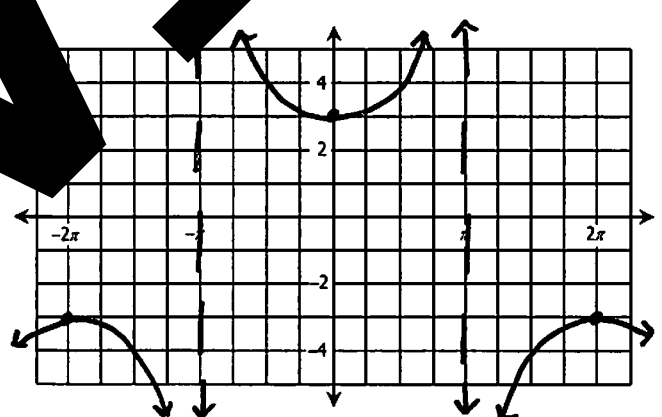
3. $f(x) = \frac{7}{2} \cdot \csc \frac{4}{3}x$

Period: $\frac{3\pi}{2}$



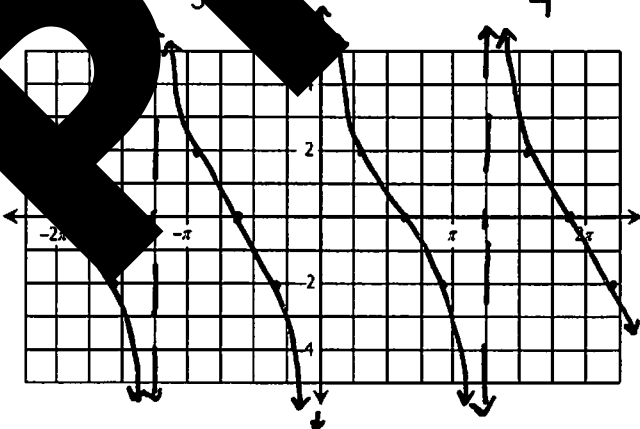
4. $f(x) = 3 \cdot \csc x$

Period: 4π



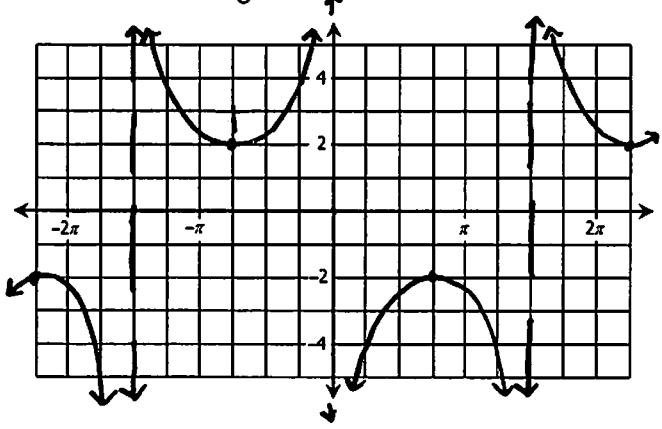
5. $f(x) = 2 \cdot \cot \frac{5}{4}x$

Period: $\frac{5\pi}{4}$



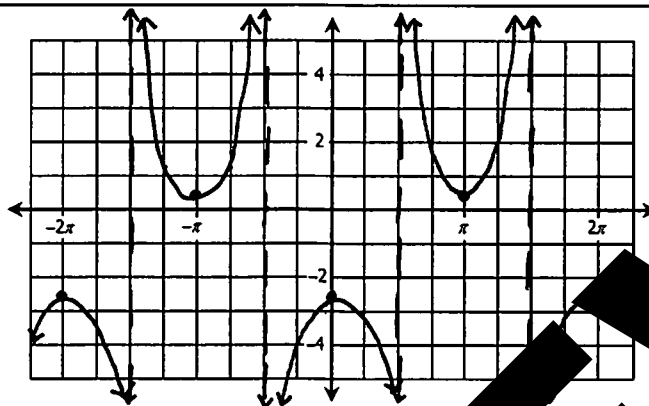
6. $f(x) = -2 \cdot \csc \frac{2}{3}x$

Period: 3π



Directions: Graph each function, showing the asymptotes on the graph. Identify the period, phase shift, and vertical shift of each function.

7. $f(x) = \frac{3}{2} \cdot \sec(x + \pi) - 1$



Period:

2

Phase

Left π

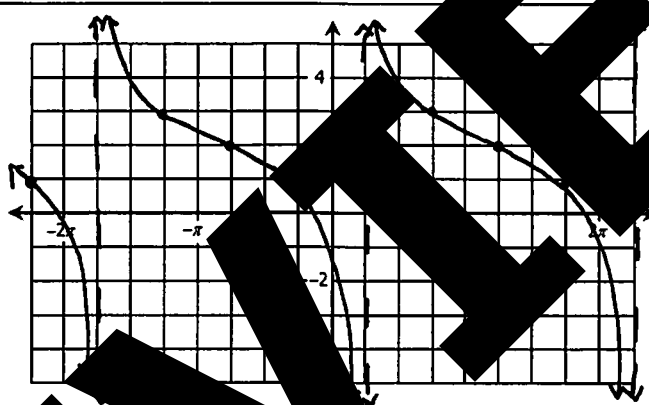
Vertical

Down 1

Midline:

$y = -1$

8. $f(x) = \cot\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right) + 2$



Period:

2 π

Phase Shift:

Right $\frac{\pi}{4}$

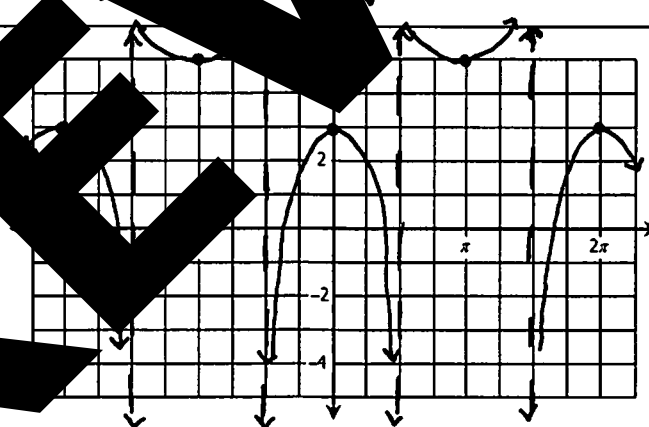
Vertical Shift:

up 2

Midline:

$y = 2$

9. $f(x) = -\csc\left(x + \frac{\pi}{2}\right) + 4$



Period:

2 π

Phase Shift:

Left $\frac{\pi}{2}$

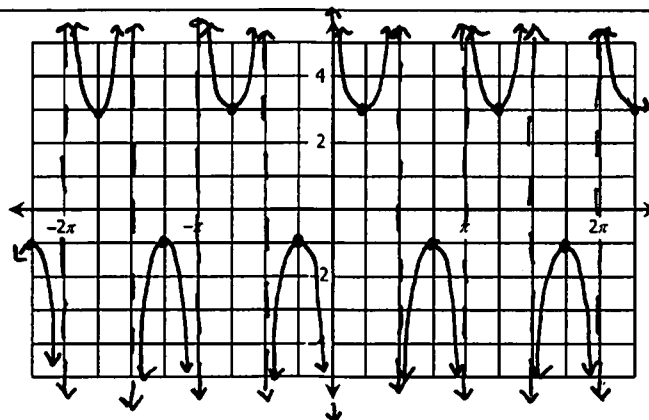
Vertical Shift:

up 4

Midline:

$y = 4$

10. $f(x) = \sec 2\left(x + \frac{\pi}{4}\right) + 1$



Period:

π

Phase Shift:

Left $\frac{3\pi}{4}$

Vertical Shift:

up 1

Midline:

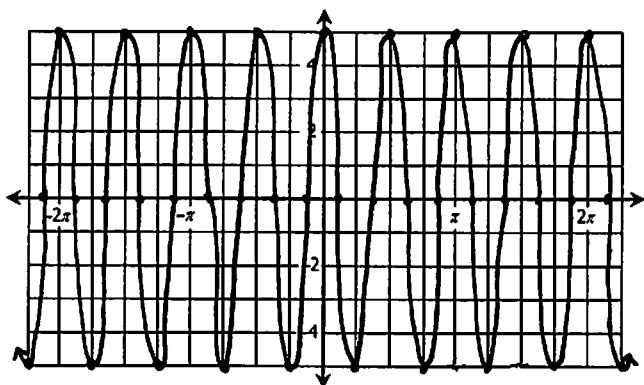
$y = 1$

GRAPHING TRIGONOMETRIC FUNCTIONS *Review!*

Set I (no shifts):

Graph the function, then identify the amplitude and period.

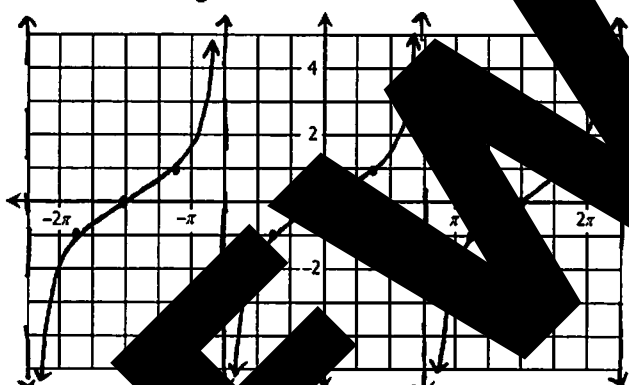
1. $f(x) = 5 \cdot \cos 4x$



Amplitude:
5

Period:
 $\frac{\pi}{2}$

2. $f(x) = \tan \frac{2}{3}x$

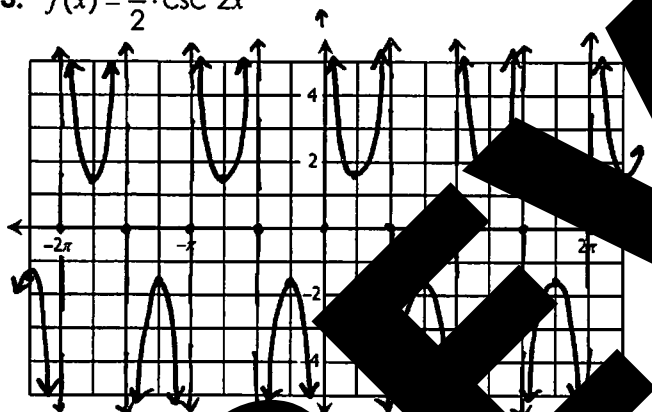


Amplitude:

undefined

Period:
 $\frac{3\pi}{2}$

3. $f(x) = \frac{3}{2} \cdot \csc 2x$



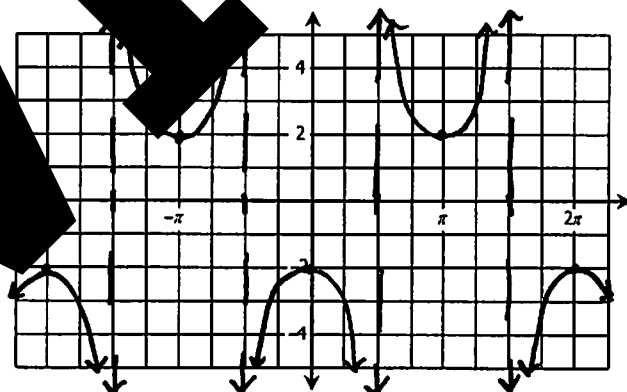
Amplitude:

undefined

Period:

π

4. $f(x) = 2 \cdot \sec x$



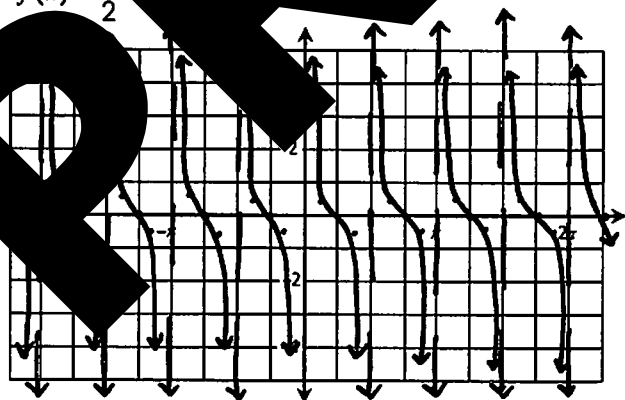
Amplitude:

undefined

Period:

2π

5. $f(x) = \frac{1}{2}$



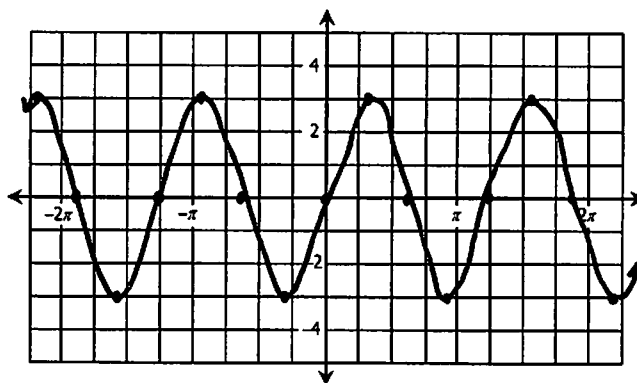
Amplitude:

undefined

Period:

$\frac{\pi}{2}$

6. $f(x) = 3 \cdot \sin \frac{8}{5}x$



Amplitude:

3

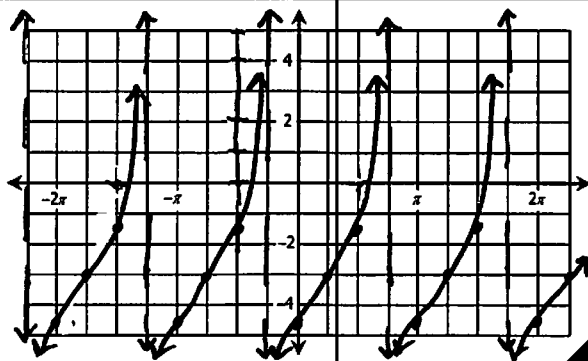
Period:

$\frac{5\pi}{4}$

Set 2 (with shifts):

Graph the function, then identify the characteristics.

7. $f(x) = \frac{3}{2} \tan \left(x - \frac{\pi}{4} \right) - 3$



Amplitude: undef

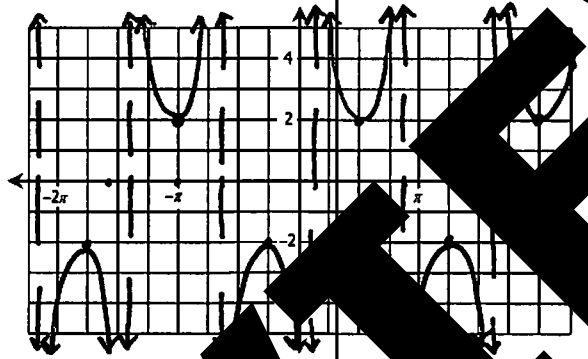
Period: π

Phase Shift: Right $\frac{\pi}{4}$

Vertical Shift: Down 3

Midline: $y = -3$

8. $f(x) = 2 \cdot \sec \frac{4}{3}(x + \pi)$



Amplitude: undef

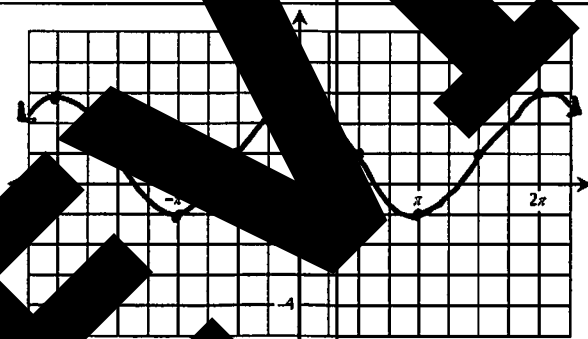
Period: $\frac{3\pi}{2}$

Phase Shift: Left π

Vertical Shift: None

Midline: $y = 0$

9. $f(x) = 2 \cdot \sin \left(x - \frac{3\pi}{2} \right) + 1$



Amplitude: 2

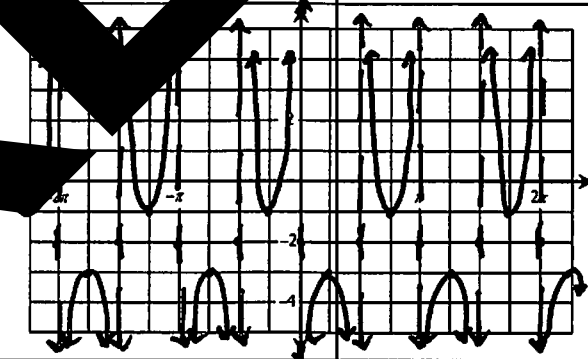
Period: 2π

Phase Shift: Right $\frac{3\pi}{2}$

Vertical Shift: Up 1

Midline: $y = 1$

10. $f(x) = \csc 2 \left(x + 2\pi \right) - 2$



Amplitude: undef

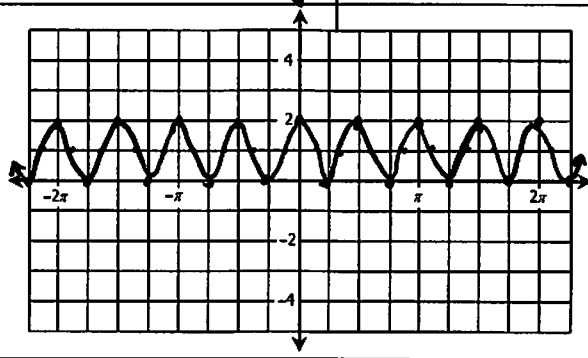
Period: π

Phase Shift: Right $\frac{\pi}{2}$

Vertical Shift: Down 2

Midline: $y = -2$

11. $f(x) = \cos 4(x + 2\pi) + 1$



Amplitude: 1

Period: $\frac{\pi}{2}$

Phase Shift: Left 2π

Vertical Shift: Up 1

Midline: $y = 1$

Name: _____

Unit 5: Trigonometric Functions



Date: _____ Per: _____

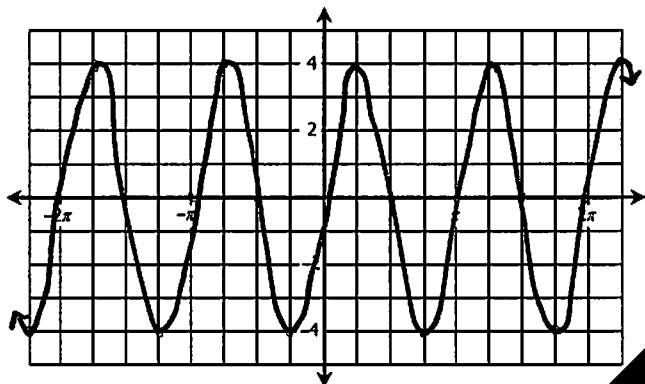
Homework 13: Graphing Trig Functions Review

**** This is a 2-page document! ****

Directions: Identify the amplitude and period, then graph the function.

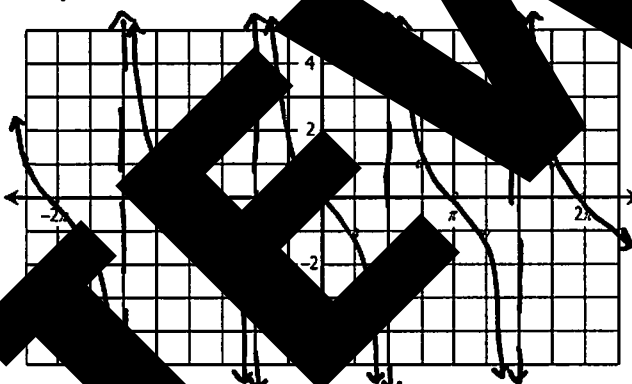
1. $f(x) = 4 \cdot \sin 2x$

Amplitude: 4 Period: π



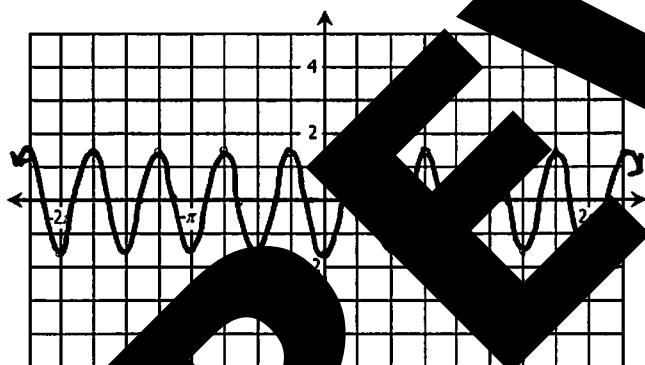
2. $f(x) = -\tan x$

Amplitude: undef Period: π



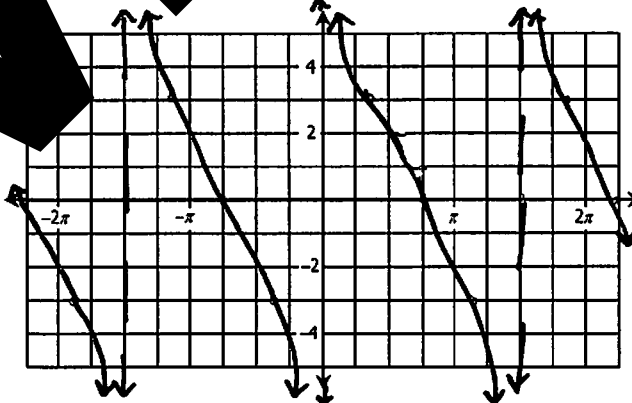
3. $f(x) = -\frac{3}{2} \cdot \cos 4x$

Amplitude: $3/2$ Period: $\pi/2$



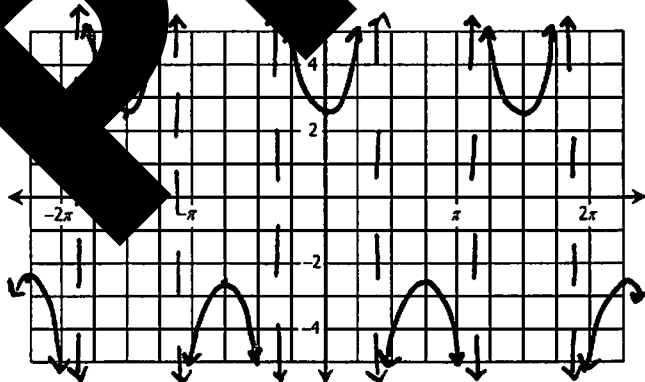
4. $f(x) = 3 \cdot \csc x$

Amplitude: undef Period: $\frac{3\pi}{2}$



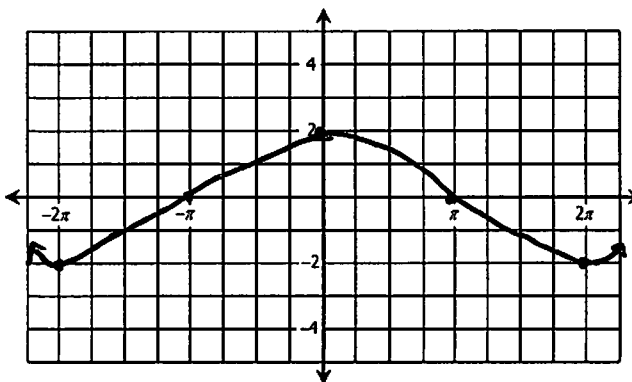
5. $f(x) = 5 \cdot \sec x$

Amplitude: undef Period: $\frac{3\pi}{2}$

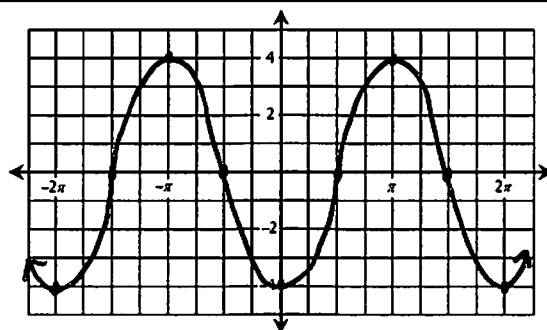


6. $f(x) = 2 \cdot \cos \frac{1}{2}x$

Amplitude: 2 Period: 4π



7. $f(x) = 4 \cdot \cos(x + \pi)$



Amplitude: 4

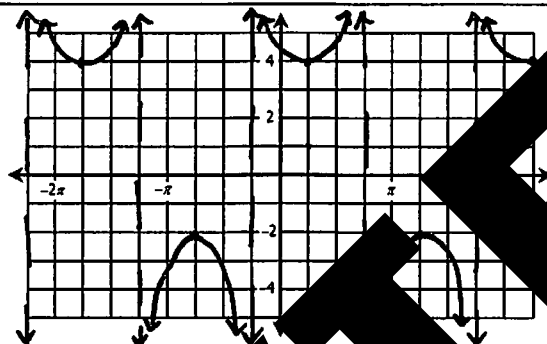
Period: 2π

Phase Shift: Left π

Vertical Shift: None

Midline: $y = 0$

8. $f(x) = 3 \cdot \sec\left(x - \frac{\pi}{4}\right) + 1$



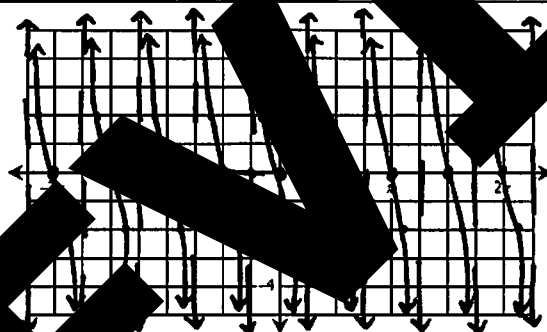
Period: 2π

Phase Shift: Right $\frac{\pi}{4}$

Vertical Shift: Up 1

Midline: $y = 1$

9. $f(x) = 2 \cdot \cot 2\left(x + \frac{\pi}{4}\right)$



Amplitude: undef

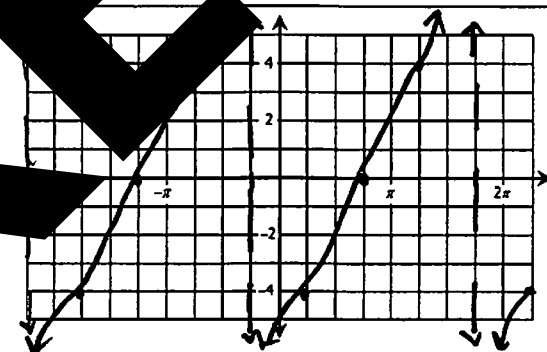
Period: $\frac{\pi}{2}$

Phase Shift: Left $\frac{\pi}{4}$

Vertical Shift: None

Midline: $y = 0$

10. $f(x) = 4 \cdot \tan \frac{1}{2}\left(x - \frac{3\pi}{4}\right)$



Amplitude: undef

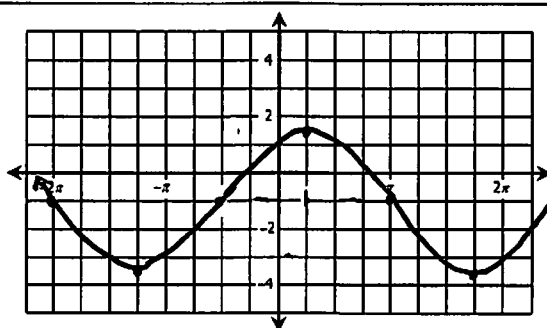
Period: 2π

Phase Shift: Right $\frac{3\pi}{4}$

Vertical Shift: None

Midline: $y = 0$

11. $f(x) = \frac{5}{2} \sin \frac{2}{3}\left(x + \frac{\pi}{2}\right) - 1$



Amplitude: $\frac{5}{2}$

Period: 3π

Phase Shift: Left $\frac{\pi}{2}$

Vertical Shift: Down 1

Midline: $y = -1$

Name: _____

Pre-Calculus

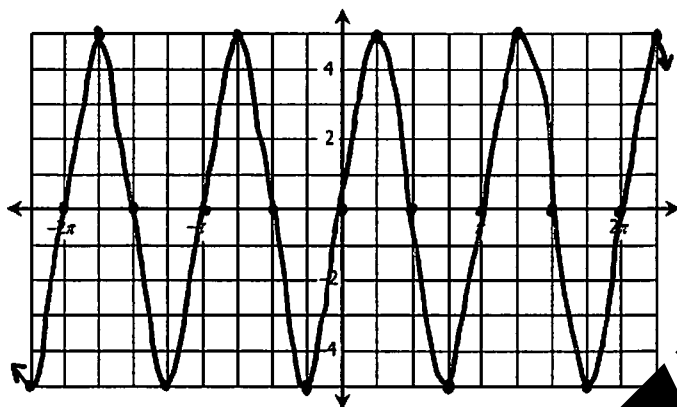
Date: _____ Per: _____

Unit 5: Trigonometric Functions

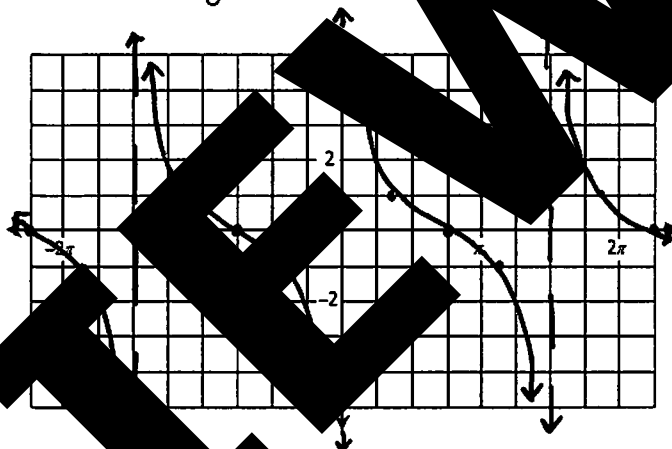
Quiz 5-4: Graphing Trigonometric Functions

Directions: Give the amplitude and period for each function. Then graph.

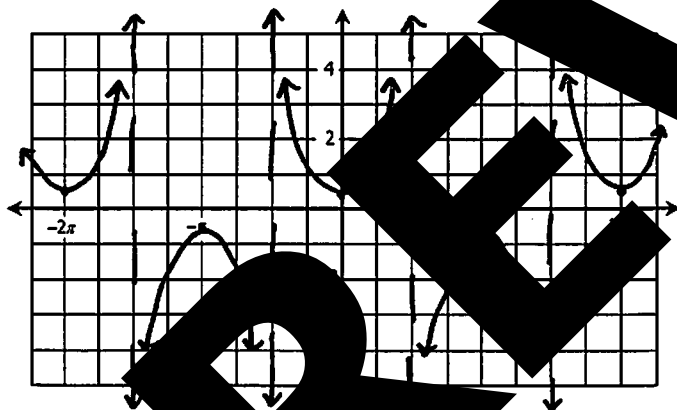
1. $f(x) = 5 \cdot \sin 2x$ Amp: 5 Per: π



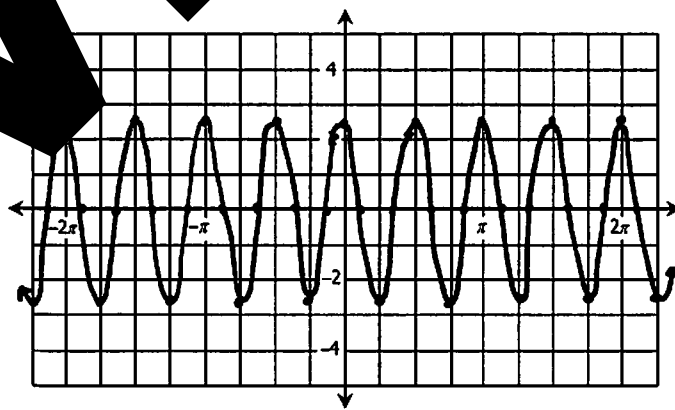
2. $f(x) = \cot \frac{2}{3}x$ Amp: _____ Per: _____



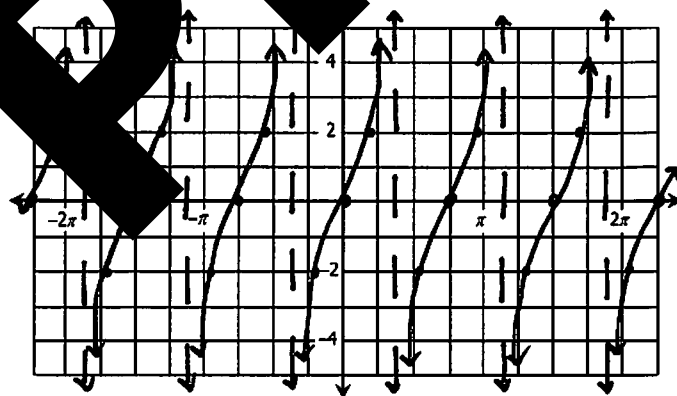
3. $f(x) = \frac{1}{2} \cdot \sec x$ Amp: Under Per: 2π



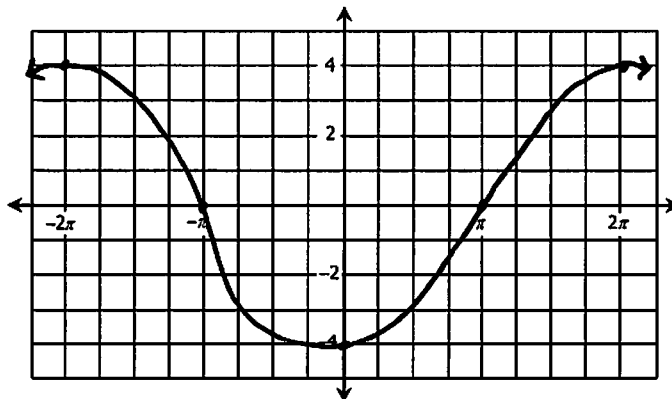
4. $f(x) = \frac{5}{2} \cdot \cos 4x$ Amp: $\frac{5}{2}$ Per: $\frac{\pi}{2}$



5. $f(x) = 2 \cdot \tan x$ Amp: Under Per: $\frac{3\pi}{4}$

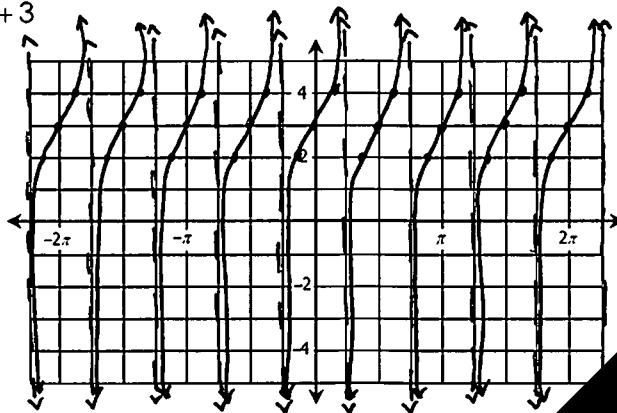


6. $f(x) = -4 \cdot \cos \frac{1}{2}x$ Amp: 4 Per: 4π



Directions: Identify the amplitude, period, translations, and midline. Then graph.

7. $f(x) = \tan 2\left(x + \frac{3\pi}{2}\right) + 3$



Amplitude: undef

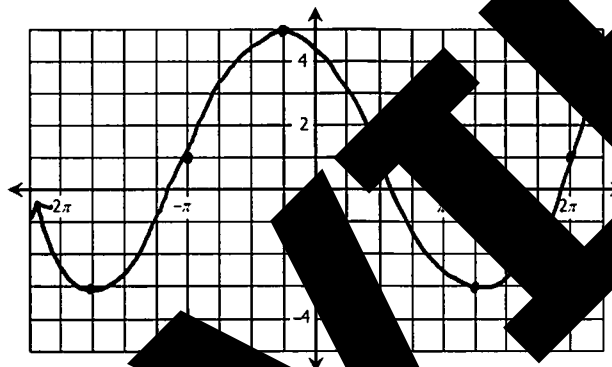
Period: pi/2

Phase Shift: Left 3pi/2

Vertical Shift: Up 3

Midline: y = 3

8. $f(x) = 4 \cdot \sin \frac{2}{3}(x - 2\pi) + 1$



Amplitude: 4

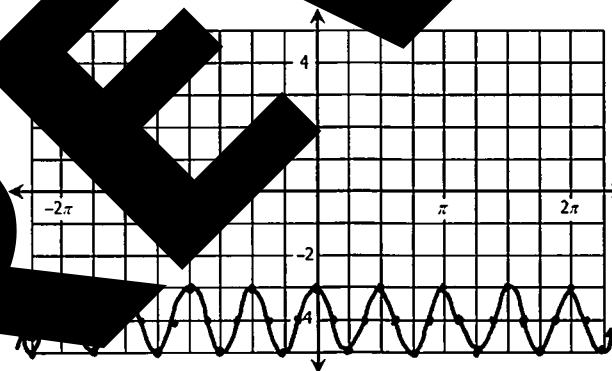
Period: 3pi

Phase Shift: Right 2pi

Vertical Shift: up 1

Midline: y = 1

9. $f(x) = \cos 4(x - \pi) - 4$



Amplitude: 1

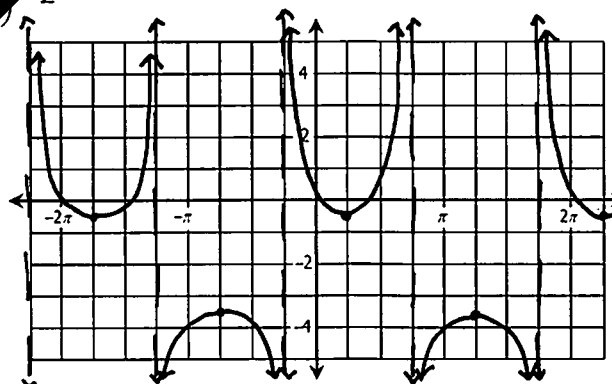
Period: pi/2

Phase Shift: Right pi

Vertical Shift: Down 4

Midline: y = -4

10. $f(x) = \frac{3}{2} \cdot \csc \left(x + \frac{\pi}{4}\right) - 2$



Amplitude: undef

Period: 2pi

Phase Shift: Left pi/4

Vertical Shift: Down 2

Midline: y = -2

Name:

Date:

Topic:

Class:

Main Ideas/Questions

Notes/Examples

INVERSE*Trigonometric*
FUNCTIONS

A function has an inverse if and only if it is a one-to-one function. (This means that it must pass the horizontal line test to have an inverse.) However, if the domain is restricted to a certain interval, the sine, cosine, and tangent functions have inverse functions.

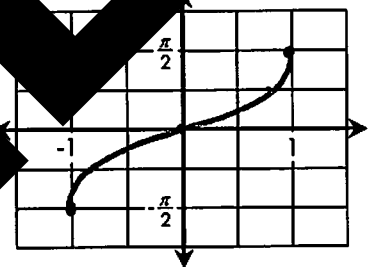
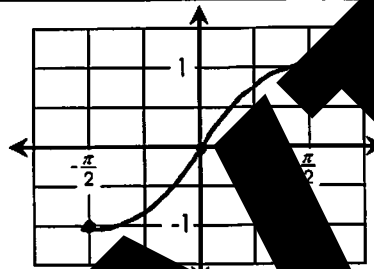
When graphed on a restricted interval:

- the inverse of the sine function is the arcsine function
- the inverse of the cosine function is the arccosine function
- the inverse of the tangent function is the arctangent function

ARCSINE*Function*

$$f(x) = \sin x; -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$f(x) = \arcsin x \text{ or } f(x) = \sin^{-1} x$$



$$D: -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

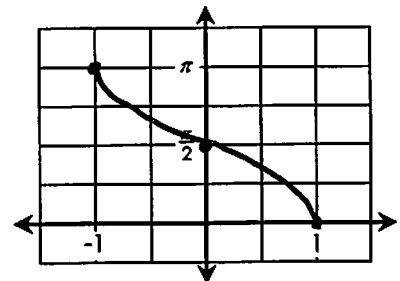
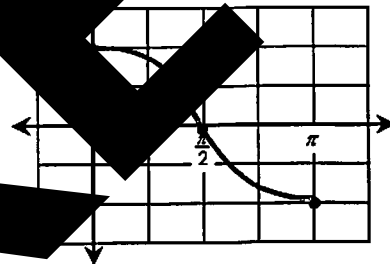
$$D: -1 \leq x \leq 1$$

$$R: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

ARCCOSINE*Function*

$$f(x) = \cos x; 0 \leq x \leq \pi$$

$$f(x) = \arccos x \text{ or } f(x) = \cos^{-1} x$$



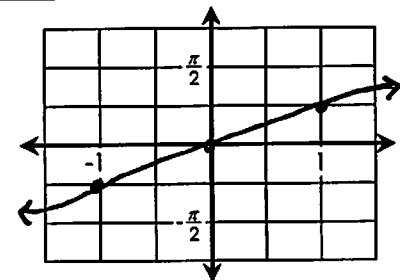
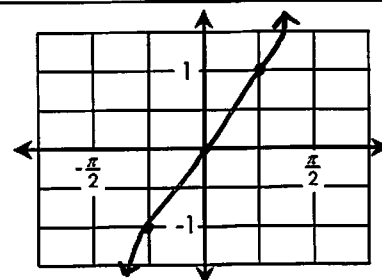
$$D: 0 \leq x \leq \pi$$

$$D: -1 \leq x \leq 1$$

$$R: 0 \leq y \leq \pi$$

$$f(x) = \tan x; -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(x) = \arctan x \text{ or } f(x) = \tan^{-1} x$$



$$D: -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$R: -\infty < y < \infty$$

$$D: -\infty < x < \infty$$

$$R: -\frac{\pi}{2} < y < \frac{\pi}{2}$$

EVALUATING INVERSE FUNCTIONS

Use the unit circle to help you evaluate inverse trigonometric functions! However, remember that the domain and range are restricted on trigonometric functions.

Directions: Find the exact values of each expression, if it exists.

1. $\arcsin 1$

$$\frac{\pi}{2}$$

2. $\sin^{-1} \frac{\sqrt{3}}{2}$

$$\frac{\pi}{3}$$

3. $\arcsin(-2)$

$$\text{DNE}$$

4. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$$\frac{3\pi}{4}$$

5. $\cos^{-1} 3$

$$\text{DNE}$$

6. $\arccos\left(\frac{\sqrt{3}}{2}\right)$

$$\frac{\pi}{6}$$

7. $\arctan \frac{\sqrt{3}}{3}$

$$\frac{\pi}{6}$$

8. $\arctan \frac{1}{\sqrt{3}}$

$$\frac{\pi}{6}$$

9. $\tan^{-1} \sqrt{3}$

$$\frac{\pi}{3}$$

APPLICATIONS

10. A ship left Port A and sailed due east to Port B. From Port B, the ship sailed 120 miles northwest and arrived at Port C. Find the bearing from Port B to the current location of the ship.



11. The angle of elevation from a football on a football field to the top of a goal post depends on d , the distance from the ball to the base of the goal post.

- a) Write θ as a function of d . $\theta = \arctan \frac{10}{d}$

b) Using your graphing calculator, find θ when d is 75 feet. Round to the nearest tenth of a degree.

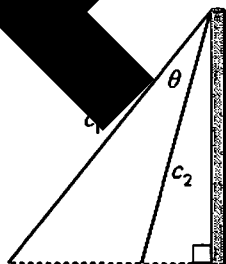
$$\theta = \arctan \frac{10}{75} \approx 7.6^\circ$$

12. Two cables are secured from the top of a 50-foot tall utility pole to the ground. The lengths of the cables are given as c_1 and c_2 .

- a) Write θ as a function of c_1 and c_2 . $\theta = \arccos \frac{50}{c_1} - \arccos \frac{50}{c_2}$

b) Using your graphing calculator, find θ if the cables are 120 feet and 65 feet long.

$$\theta = \arccos \frac{50}{120} - \arccos \frac{50}{65} \approx 25.7^\circ$$



Name:

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Main Ideas/Questions

Notes/Examples

Transformations of INVERSE TRIG FUNCTIONS

One strategy to graph transformations of inverse function is to find the inverse function, create a table of values, then invert the values and graph.

Example 1: Graph $f(x) = \arcsin(2x)$

Step 1:

Find the inverse of the function.

$$y = \arcsin(2x)$$

↓

$$x = \arcsin(2y)$$

$$\sin x = 2y$$

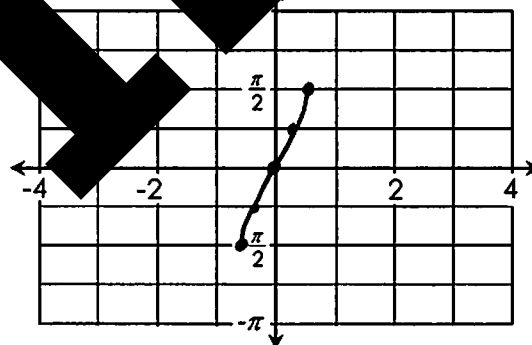
$$\frac{1}{2} \sin x = y$$

$$f^{-1}(x) = \frac{1}{2}x$$

Step 2: Make a table of values.

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$f^{-1}(x)$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$

Step 3: Graph the inverted points.



Note: This shifts the range down π , therefore the range is now $[-\pi, 0]$.

Example 2: Graph $f(x) = \cos^{-1}x - \pi$

Step 1:

Find the inverse of the function.

$$y = \cos^{-1}x - \pi$$

↓

$$x = \cos^{-1}y - \pi$$

$$x + \pi = \cos^{-1}y$$

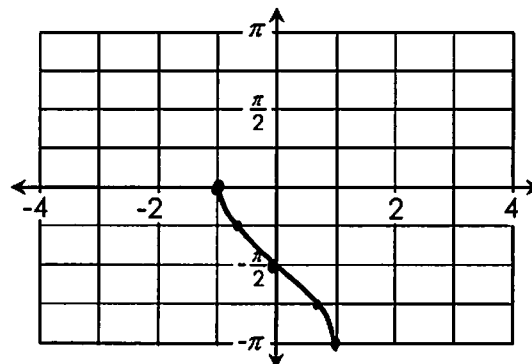
$$\cos(x + \pi) = y$$

$$f^{-1}(x) = \cos(x + \pi)$$

Step 2: Make a table of values.

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$f^{-1}(x)$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1

Step 3: Graph the inverted points.



Example 3:

$$f(x) = \arctan(x+2)$$

$$y = \tan^{-1}(x+2)$$

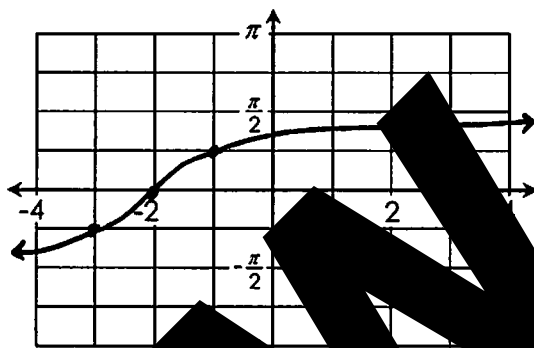
$$x = \tan^{-1}(y+2)$$

$$\tan x = y+2$$

$$\tan x - 2 = y$$

$$f'(x) = \tan x - 2$$

x	$f'(x)$
$-\frac{\pi}{2}$	undef
$-\frac{\pi}{4}$	-3
0	-2
$\frac{\pi}{4}$	-1
$\frac{\pi}{2}$	undef



Example 4:

$$f(x) = \cos^{-1} \frac{1}{4}x$$

$$y = \cos^{-1} \frac{1}{4}x$$

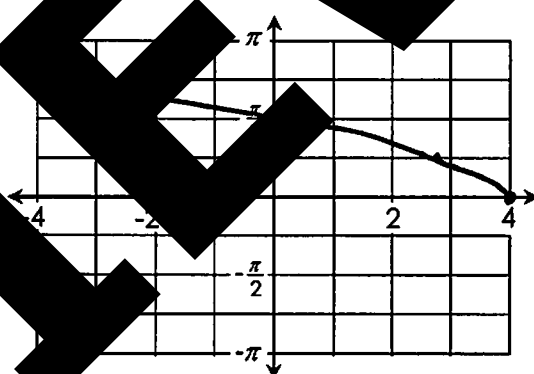
$$x = \cos^{-1} \frac{1}{4}y$$

$$\cos x = \frac{1}{4}y$$

$$4 \cos x = y$$

$$f'(x) = 4$$

x	$f'(x)$
0	4
$\frac{\pi}{4}$	4
$\frac{\pi}{2}$	$-2\sqrt{2}$
$\frac{3\pi}{4}$	-4



Example 5:

$$f(x) = \arcsin x + \frac{\pi}{2}$$

$$y = \arcsin x + \frac{\pi}{2}$$

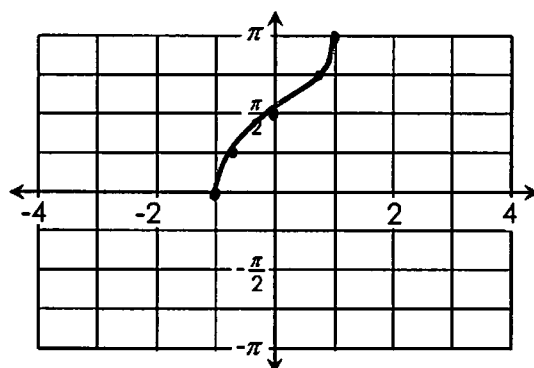
$$x = \arcsin(y - \frac{\pi}{2})$$

$$x - \frac{\pi}{2} = \arcsin y$$

$$\sin(x - \frac{\pi}{2}) = y$$

$$f'(x) = \sin(x - \frac{\pi}{2})$$

x	$f'(x)$
0	-1
$\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$
π	1



$$f(x) = \tan^{-1} 3(x-1)$$

$$y = \tan^{-1} 3(x-1)$$

$$x = \tan^{-1} 3(y-1)$$

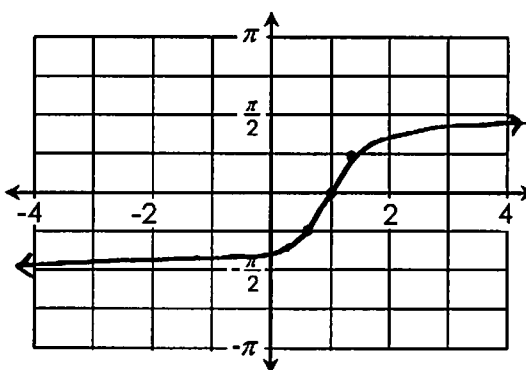
$$\tan x = 3(y-1)$$

$$\frac{1}{3} \tan x = y-1$$

$$\frac{1}{3} \tan x + 1 = y$$

$$f'(x) = \frac{1}{3} \tan x + 1$$

x	$f'(x)$
$-\frac{\pi}{2}$	undef
$-\frac{\pi}{4}$	$\frac{2}{3}$
0	1
$\frac{\pi}{4}$	$\frac{4}{3}$
$\frac{\pi}{2}$	undef



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Unit 5: Trigonometric Functions

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Homework 14: Inverse Trigonometric Functions

**** This is a 2-page document! ******Directions:** Find the exact values of each expression, if it exists.

1. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

$\frac{5\pi}{6}$

2. $\tan^{-1} 1$

$\frac{\pi}{4}$

3. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$-\frac{\pi}{4}$

4. $\sin^{-1}\left(-\frac{1}{2}\right)$

$-\frac{\pi}{6}$

5. $\arccos(2)$

DNE

6. $\arctan(-\sqrt{3})$

$-\frac{\pi}{3}$

7. $\sin^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{6}$

8. $\arctan\left(-\frac{1}{3}\right)$

$-\frac{\pi}{6}$

9. Molly is looking up at the the St. Louis Arch which stands 630 feet high. The angle of elevation, θ , from her line of sight to the top of the monument depends on the distance, d , she is from the base. If Molly's eyes are 5 feet 3 inches from the ground, write a function for θ in terms of d .

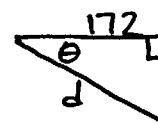


$\theta = \arctan \frac{630 - 5.75}{d}$

$\theta = \arctan \frac{624.25}{d}$

10. A boat leaves the dock and travels 172 meters before dropping the anchor. The angle of depression depends on the straightline distance between the end of the dock and the anchor.

a) Write θ as a function of d . $\theta = \arccos \frac{172}{d}$



- b) Using your calculator, find θ when d is 205 meters. Round to the nearest tenth of a degree.

$\theta = \arccos \frac{172}{205} = 33^\circ$

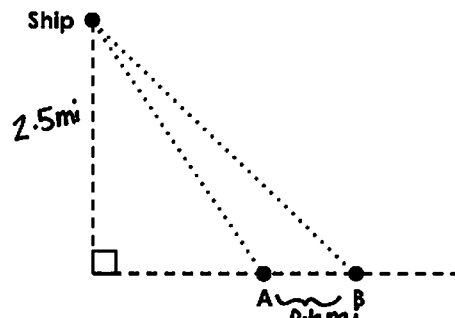
11. The captain on a ship picks up two shipwrecks, A and B, 2.5 miles beneath the surface. The distance between the shipwrecks is 0.6 miles. The angle of depression, θ , to Shipwreck B depends on the horizontal distance, d , from the ship to Shipwreck A.

Write a function for θ in terms of d .

$\theta = \arctan \frac{2.5}{d + 0.6}$

- b) Find θ if d is 1.4 miles.

$\theta = \arctan \frac{2.5}{1.4 + 0.6} = 51.3^\circ$



Directions: Graph each of the following.

12. $f(x) = \arctan 2x$

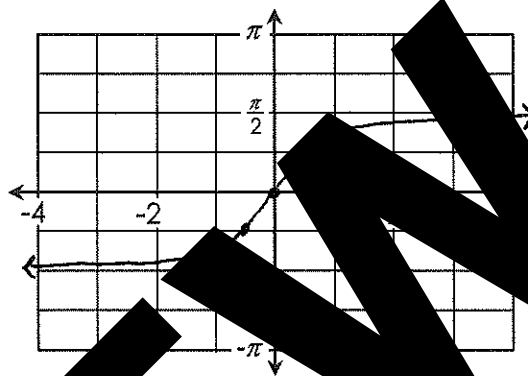
$$x = \tan^{-1} 2y$$

$$\tan x = 2y$$

$$\frac{1}{2} \tan x = y$$

$$f^{-1}(x) = \frac{1}{2} \tan x$$

x	$f^{-1}(x)$
$-\frac{\pi}{2}$	undef.
$-\frac{\pi}{4}$	$-\frac{1}{2}$
0	0
$\frac{\pi}{4}$	$\frac{1}{2}$
$\frac{\pi}{2}$	undef.



13. $f(x) = \cos^{-1} x - \frac{3\pi}{4}$

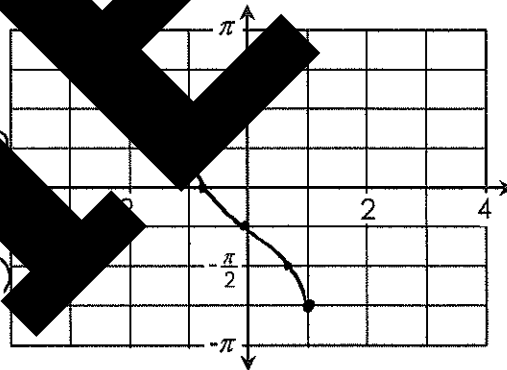
$$x = \cos^{-1} y - \frac{3\pi}{4}$$

$$x + \frac{3\pi}{4} = \cos^{-1} y$$

$$\cos(x + \frac{3\pi}{4}) = y$$

$$f^{-1}(x) = \cos(x + \frac{3\pi}{4})$$

x	$f^{-1}(x)$
$-\frac{3\pi}{4}$	1
$-\frac{\pi}{2}$	$\frac{1}{2}$
0	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	0



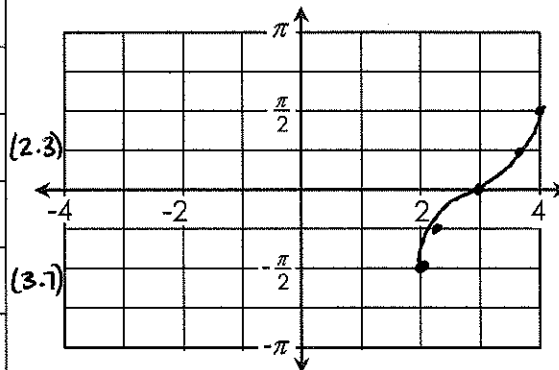
14. $f(x) = \arcsin(x-3)$

$$x = \sin^{-1}(y-3)$$

$$\sin x = y-3$$

$$\sin x + 3 = y$$

x	$f^{-1}(x)$
$-\frac{\pi}{2}$	2
$-\frac{\pi}{4}$	$\frac{6-\sqrt{2}}{2}$
0	3
$\frac{\pi}{4}$	$\frac{6+\sqrt{2}}{2}$
$\frac{\pi}{2}$	4



$$y = \sin x + 3$$

$$f^{-1}(x) = \sin^{-1} \frac{1}{2}(x+1) + \frac{\pi}{2}$$

$$\sin^{-1} \frac{1}{2}(y+1) + \frac{\pi}{2}$$

$$x = \sin^{-1} \frac{1}{2}(y+1) + \frac{\pi}{2}$$

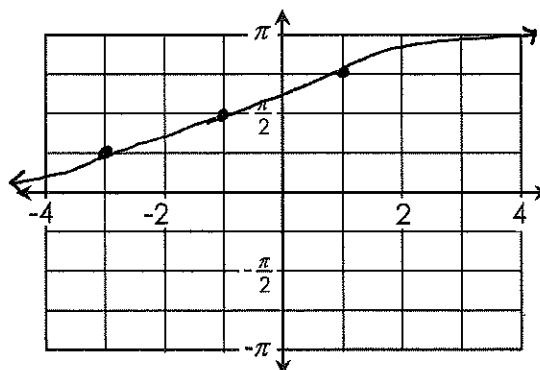
$$\tan(x - \frac{\pi}{2}) = \frac{1}{2}(y+1)$$

$$2 \tan(x - \frac{\pi}{2}) = y+1$$

$$2 \tan(x - \frac{\pi}{2}) - 1 = y$$

$$f^{-1}(x) = 2 \tan(x - \frac{\pi}{2}) - 1$$

x	$f^{-1}(x)$
0	undef.
$\frac{\pi}{4}$	-3
$\frac{\pi}{2}$	-1
$\frac{3\pi}{4}$	1
π	undef.



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Main Ideas/Questions	Notes/Examples
COMPOSITIONS of Trigonometric Functions	Recall: Given function f and it's inverse f^{-1}, if x is in the domain of both functions, then $f[f^{-1}(x)] = x \quad \text{and} \quad f^{-1}[f(x)] = x$ The properties of compositions of trigonometric functions are listed below: If $-1 \leq x \leq 1$, then $\sin(\sin^{-1} x) = x$ If $-1 \leq x \leq 1$, then $\cos(\cos^{-1} x) = x$ If $-\infty < x < \infty$, then $\tan(\tan^{-1} x) = x$ If $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then $\sin^{-1}(\sin x) = x$ If $0 < x < \pi$, then $\cos^{-1}(\cos x) = x$ If $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then $\tan^{-1}(\tan x) = x$ Directions: Find the exact value of each expression, if it exists.
	1. $\sin\left[\sin^{-1}\left(-\frac{1}{3}\right)\right] = -\frac{1}{3}$ 2. $\cos\left(\arcsin\left(\frac{1}{5}\right)\right) = \frac{4}{5}$ 3. $\cos\left(\cos^{-1}\frac{3}{2}\right) = \text{DNE}$
	4. $\tan\left[\tan^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{2}$ 5. $\tan^{-1}\left(\tan\left(\frac{1}{4}\right)\right) = \frac{1}{4}$ 6. $\sin\left[\arcsin\left(-\frac{7}{6}\right)\right] = \text{DNE}$
	7. $\sin\left(\sin^{-1}\left(\frac{1}{4}\right)\right) = \frac{1}{4}$ 8. $\cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$
	9. $\tan^{-1}\left[\tan\left(-\frac{\pi}{2}\right)\right] = \text{DNE}$ 10. $\sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$
	11. $\arcsin\left(\sin\frac{3\pi}{4}\right) = \arcsin\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$ 12. $\tan^{-1}(\tan \pi) = \tan^{-1}(\tan 0) = 0$
	13. $\sin^{-1}\left(\sin\frac{11\pi}{6}\right) = \sin^{-1}\left(\sin-\frac{\pi}{6}\right) = -\frac{\pi}{6}$ 14. $\arccos\left[\cos\left(-\frac{4\pi}{3}\right)\right] = \arccos\left[\cos\left(\frac{2\pi}{3}\right)\right] = \frac{2\pi}{3}$

Note: The angle θ is in the domain, so we can use it using an equivalent trig value that is in the domain.

MIXING FUNCTIONS

SET 1: Unit Circle Values

Find the exact value of each expression, if it exists. Work inside out!

15. $\cos^{-1}\left(\sin \frac{\pi}{6}\right)$

$\cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$

16. $\arctan(\cos 0)$

$\arctan(1) = \boxed{\frac{\pi}{4}}$

17. $\sin^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$

$\sin^{-1}\left[\tan\left(\frac{\pi}{4}\right)\right] = \sin^{-1}(1) = \boxed{\frac{\pi}{2}}$

18. $\arcsin\left(\cos \frac{5\pi}{6}\right)$

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

19. $\cos^{-1}\left(\tan \frac{\pi}{4}\right)$

$\cos^{-1}(1) = \boxed{0}$

$\arctan\left[\sin\left(-\frac{\pi}{2}\right)\right]$

$\arctan(-1) = \frac{7\pi}{4} = \boxed{-\frac{\pi}{4}}$

21. $\cos\left(\sin^{-1} \frac{1}{2}\right)$

$\cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

22. $\sin\left(\arccos \frac{1}{2}\right)$

$\sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

23. $\sin(\arccos 1)$

$\sin(0) = \boxed{0}$

24. $\cos(\tan^{-1} \sqrt{3})$

$\cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$

SET 2

Non-Unit Circle Values

- "inside" angle θ .
- Determine which quadrant θ must lie.
- Draw a right triangle and find the missing side.
- Evaluate the outside trigonometric function. (Keep the quadrant in mind.)

25. $\cos^{-1}\left(\frac{8}{17}\right)$

$\cos$ is + in Q1+Q4
 $\cos^{-1}$ is only in Q1+Q2
 $\therefore \theta$ is in Q1

let $\theta = \cos^{-1}\left(\frac{8}{17}\right) \rightarrow \cos \theta = \frac{8}{17}$

$8^2 + b^2 = 17^2$
 $b^2 = 225$
 $b = 15$

$\tan \theta = \frac{15}{8}$



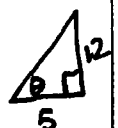
26. $\sin\left[\arctan\left(-\frac{12}{5}\right)\right]$

$\tan$ is - in Q2+Q4
 $\tan^{-1}$ is only in Q1+Q4
 $\therefore \theta$ is in Q4.

let $\theta = \arctan\left(-\frac{12}{5}\right) \rightarrow \tan \theta = -\frac{12}{5}$

$5^2 + 12^2 = c^2$
 $169 = c^2$
 $c = 13$

$\sin \theta = -\frac{12}{13}$



Name: _____

Unit 5: Trigonometric Functions

Date: _____ Per: _____

Homework 15: Compositions of Trigonometric Functions

Directions: Find the exact value of each expression, if it exists.

1. $\cos\left(\arccos \frac{4}{5}\right)$

$\boxed{\frac{4}{5}}$

2. $\tan\left(\tan^{-1} \frac{7}{3}\right)$

$\boxed{\frac{7}{3}}$

3. $\sin\left[\sin^{-1}\left(-\frac{5}{7}\right)\right]$

$\boxed{-\frac{5}{7}}$

4. $\tan^{-1}\left(\tan \frac{\pi}{3}\right)$

$\boxed{\frac{\pi}{3}}$

5. $\cos^{-1}(\cos \pi)$

$\boxed{\pi}$

6. $\arcsin\left[\sin\left(-\frac{7\pi}{4}\right)\right]$

$\boxed{-\frac{\pi}{4}}$

7. $\arctan\left(\tan \frac{5\pi}{6}\right)$

$= \arctan\left[\tan\left(-\frac{\pi}{6}\right)\right] = \boxed{-\frac{\pi}{6}}$

8. $\arccos\left(\cos \frac{3\pi}{2}\right)$

$= \arccos(\cos \pi) = \boxed{\pi}$

9. $\arcsin\left[\sin\left(-\frac{5\pi}{3}\right)\right]$

$= \arcsin\left[\sin\left(\frac{\pi}{3}\right)\right] = \boxed{\frac{\pi}{3}}$

10. $\cos\left(\arcsin \frac{\sqrt{3}}{2}\right)$

$\cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$

11. $\tan\left(\arccos \frac{\sqrt{2}}{2}\right)$

$\tan\left(\frac{\pi}{4}\right) = \boxed{1}$

12. $\cos^{-1}\left[\sin\left(-\frac{11\pi}{6}\right)\right]$

$\cos^{-1}\left(\sin \frac{\pi}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$

13. $\sin\left(\tan^{-1} \frac{\sqrt{3}}{3}\right)$

$\sin \frac{\pi}{6} = \boxed{\frac{1}{2}}$

14. $\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$

$\cos^{-1}(-1) = \boxed{\pi}$

15. $\sin[\arctan(-1)]$

$= \sin\left(-\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

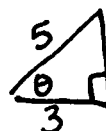
16. $\sin\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]$

$\theta = \cos^{-1}\left(-\frac{3}{5}\right) \rightarrow \cos \theta = -\frac{3}{5}$

$3^2 + b^2 = 5^2$

$b^2 = 16$

$b = 4$



$\sin \theta = \frac{4}{5}$

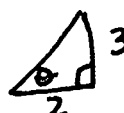
17. $\cos\left(\tan^{-1} \frac{3}{2}\right)$

$\text{let } \theta = \arctan \frac{3}{2} \rightarrow \tan \theta = \frac{3}{2}$

$2^2 + 3^2 = c^2$

$13 = c^2$

$c = \sqrt{13}$



$\cos \theta = \frac{2\sqrt{13}}{13}$

Unit 5 Test Study Guide (Trigonometric Functions)

Name: _____

Date: _____ Per: _____

Topic 1: Angles & Angle Measures

Directions: Change the degrees to radians, and the radians to degrees.

1. 310°

$$310 \cdot \frac{\pi}{180}$$

$$= \boxed{\frac{31\pi}{18}}$$

2. -155°

$$-155 \cdot \frac{\pi}{180}$$

$$= \boxed{-\frac{31\pi}{36}}$$

3. $\frac{7\pi}{3}$

$$\frac{7\pi}{3} \cdot \frac{180}{\pi}$$

$$= \boxed{420}$$

4. $-\frac{4\pi}{3}$

$$-\frac{4\pi}{3} \cdot \frac{180}{\pi}$$

$$= \boxed{-240}$$

Directions: State the quadrant in which the terminal side lies, giving two coterminal angles, one positive and one negative, for each angle.

5. -285°

Quadrant I

$$-285 + 360 = \boxed{75^\circ}$$

$$-285 - 360 = \boxed{-645^\circ}$$

6. $\frac{7\pi}{5}$

Quadrant III

$$-\frac{3\pi}{2} + 2\pi = \frac{7\pi}{5}$$

$$-\frac{3\pi}{2} - 2\pi = \boxed{-\frac{13\pi}{5}}$$

Directions: Write each angle measure in Degree-Minute-Second (DMS) form.

7. 56.875°

$$.875(60) = 52.5$$

$$.5(60) = 30$$

$$\boxed{56^\circ 32' 30''}$$

8. -207.143°

$$.143(60) = 8.58$$

$$.58(60) = 34.8$$

$$\boxed{-207^\circ 8' 35''}$$

Directions: Write each angle measure in decimal degree form.

9. $-184^\circ 16'$

$$16/60 = .26$$

$$49/3600$$

$$\boxed{-184.2803}$$

10. $78^\circ 37' 3''$

$$37/60 = .6167$$

$$3/3600 = .0008$$

$$\boxed{78.6175^\circ}$$

Topic 2: Lengths, Area of Sectors, & Circular Motion

Directions: Find the length of the intercepted arc. Round to the nearest tenth.

11. $r = 8.5$ ft

$$S = 8.5 \left(\frac{5\pi}{6} \right)$$

$$= \boxed{22.3 \text{ ft}}$$

12. $120^\circ; r = 6$ in

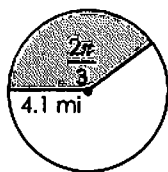
$$\hookrightarrow \frac{2\pi}{3}$$

$$S = 6 \left(\frac{2\pi}{3} \right)$$

$$= \boxed{12.6 \text{ in}}$$

Directions: Find the area of each sector. Round to the nearest tenth.

13.

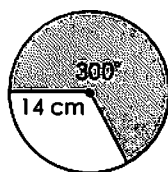


$$A = \frac{1}{2} \left(\frac{2\pi}{3} \right) (4.1)^2$$

$$= \frac{\pi}{3} (16.81)$$

$$= \boxed{17.6 \text{ mi}^2}$$

14.



$$A = \frac{1}{2} \left(\frac{5\pi}{3} \right) (14)^2$$

$$= \frac{5\pi}{6} (196)$$

$$= \boxed{161 \text{ cm}^2}$$

For questions 15 and 16: An 18-inch bicycle tire is making 140 revolutions per minute.

15. Find the linear speed in miles per hour.

$$S = 2\pi(9)(140) = 2520\pi \text{ in} ; \quad v = \frac{2520\pi \text{ in}}{1 \text{ min}}$$

$$\frac{2520\pi \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} = \boxed{7.8 \text{ mi/hr}}$$

16. Find the angular speed of the bicycle tire in radians per minute.

$$\theta = 2\pi(140) = 280\pi \text{ radians}$$

$$\omega = \frac{280\pi \text{ rad}}{1 \text{ min}} \approx \boxed{879.6 \text{ rad/min}}$$

For questions 17 and 18: A CD with a radius of 6 centimeters rotates at a speed of 237 revolutions per minute.

17. Find the linear speed in meters per second.

$$S = 2\pi(6)(237) = 2844\pi \text{ cm} ; \quad v = \frac{2844\pi \text{ cm}}{1 \text{ min}}$$

$$\frac{2844\pi \text{ cm}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} \approx \boxed{1.5 \text{ m/sec}}$$

18. Find the angular speed in radians per minute.

$$\theta = 2\pi(237) = 474\pi \text{ radians}$$

$$\omega = \frac{474\pi \text{ rad}}{1 \text{ min}} \approx \boxed{1489.1 \text{ rad/min}}$$

Topic 3: Trigonometric Functions & Solving Right Triangles

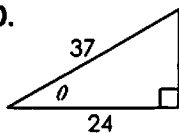
Directions: Find the missing side or angle measure. Round answers to the nearest tenth.



$$\tan 64 = \frac{23}{x}$$

$$\boxed{x = 11.2}$$

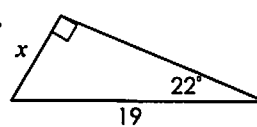
20.



$$\cos \theta = \frac{24}{37}$$

$$\boxed{\theta = 49.6^\circ}$$

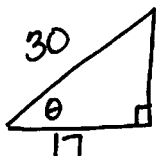
21.



$$\sin 22 = \frac{x}{19}$$

$$\boxed{x = 7.1}$$

22. A 30-foot ladder is leaning against an office building. If the base of the ladder is 17 feet from the base of the building, find the angle of elevation.

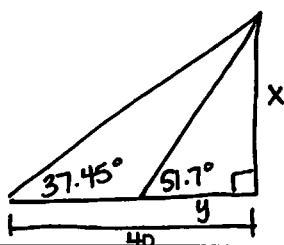


$$\cos \theta = \frac{17}{30}$$

$$\theta = \cos^{-1}\left(\frac{17}{30}\right)$$

$$\theta = 50^\circ$$

23. Sharon is bird watching and spots an eagle in a tree. When standing 40 meters from the base of the tree, the angle of elevation from the point in which she is standing is $37^\circ 2'$. After moving closer to the tree, Sharon now measures a $51^\circ 42'$ angle of elevation. Find the distance Sharon traveled between viewings.



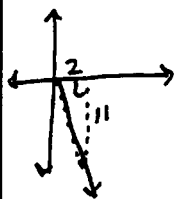
$$\tan 37.45 = \frac{x}{40} \quad x = 30.6$$

$$\tan 51.7 = \frac{x}{y} \quad y = 20.2$$

Sharon traveled $40 - 20.2 = 19.8$ ft between viewings.

Topic 4: Trigonometric Functions for Any Angle: The Unit Circle

24. $A(2, -11)$ is a point on the terminal side of θ in standard form. Find the exact values of the trigonometric functions of θ .



$$2^2 + 11^2 = r^2$$

$$125 = r^2$$

$$r = 5\sqrt{5}$$

$$y = -11$$

$$r = 5\sqrt{5}$$

$\sin \theta = \frac{-11}{5\sqrt{5}} = -\frac{11\sqrt{5}}{25}$	$\csc \theta = -\frac{5\sqrt{5}}{11}$
$\cos \theta = \frac{2}{5\sqrt{5}} = \frac{2\sqrt{5}}{25}$	$\sec \theta = \frac{5\sqrt{5}}{2}$
$\tan \theta = \frac{-11}{2}$	$\cot \theta = -\frac{2}{11}$

25. If $\cos \theta = \frac{6}{7}$ and $\theta < 0$, find the exact values of each of the trigonometric functions of θ .

$$\left. \begin{array}{l} \cos < 0 \\ \sin < 0 \end{array} \right\} \text{Q3}$$

$$6^2 + b^2 = 7^2$$

$$b^2 = 13$$

$$b = \sqrt{13}$$

$\sin \theta = -\frac{\sqrt{13}}{7}$	$\csc \theta = -\frac{7\sqrt{13}}{13}$
$\cos \theta = -\frac{6}{7}$	$\sec \theta = -\frac{7}{6}$
$\tan \theta = \frac{\sqrt{13}}{6}$	$\cot \theta = \frac{6\sqrt{13}}{13}$

Directions: Find the reference angle for each of the following.

26. 164°

$$16^\circ$$

27. -461°

$$79^\circ$$

28. $-\frac{3\pi}{4}$

$$\frac{\pi}{4}$$

29. $\frac{13\pi}{9}$

$$\frac{4\pi}{9}$$

Directions: Give the exact value of each trigonometric function.

30. $\sin\left(-\frac{4\pi}{3}\right)$

$\frac{\sqrt{3}}{2}$

31. $\sec\left(-\frac{7\pi}{6}\right)$

$-\frac{2\sqrt{3}}{3}$

32. $\cos\frac{7\pi}{4}$

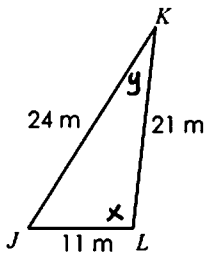
$\frac{\sqrt{2}}{2}$

33. $\cot\frac{\pi}{6}$

$\sqrt{3}$

Topic 5: Law of Sines & Law of Cosines

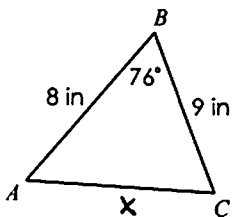
34. Solve the triangle below. Round all answers to the nearest tenth.



$$\begin{aligned} 24^2 &= 11^2 + 21^2 - 2(11)(21)\cos x \\ 576 &= 562 - 462\cos x \\ 14 &= -462\cos x \\ -.0303 &= \cos x \\ x &= 91.7^\circ \end{aligned}$$

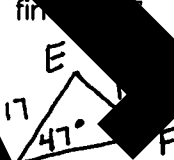
$m\angle K = 21.3^\circ$
 $m\angle L = 91.7^\circ$

35. Find AC.



$$\begin{aligned} x^2 &= 8^2 + 9^2 - 2(8)(9)\cos 76^\circ \\ x^2 &= 145 - 144\cos 76^\circ \\ x^2 &= 110 \\ x &= 10.5 \text{ in} \end{aligned}$$

36. Find DE, $m\angle D$, and $m\angle E$. $DE = 17$ m, $EF = 16$ m.



$$\begin{aligned} \frac{\sin 47^\circ}{16} &= \frac{\sin x}{17} \\ \sin x &= .7771 \\ x &= 51^\circ, 129^\circ \end{aligned}$$

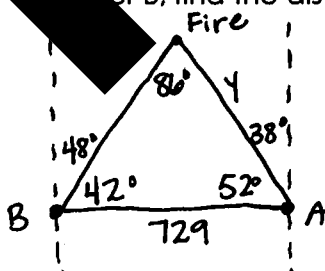
37. The log ride at an amusement park has a single hill with an incline of 127 feet and a decline of 93 feet. If the total horizontal distance between the start of the incline and end of the decline is 132 feet, find the height of the ride.



$$\begin{aligned} 127^2 &= 132^2 + y^2 - 2(132)(y)\cos x \\ 8641 &= 33553 - 33528\cos x \\ 24904 &= -33528\cos x \\ .7428 &= \cos x \\ x &= 42^\circ \end{aligned}$$

$$\begin{aligned} \sin 42^\circ &= \frac{y}{127} \\ y &= 85 \text{ ft} \end{aligned}$$

Two towers at a national park spot a forest fire. Tower A spots the fire at a bearing of N 38° W and Tower B spots the fire at a bearing of N 48° E. If Tower A is 729 meters directly east of Tower B, find the distance from each tower to the fire.

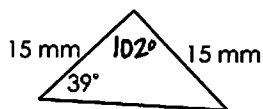


$$\begin{aligned} \frac{\sin 86^\circ}{729} &= \frac{\sin 52^\circ}{x} \\ x &= 575.9 \text{ m} \end{aligned}$$

$$\begin{aligned} \frac{\sin 86^\circ}{729} &= \frac{\sin 42^\circ}{y} \\ y &= 489 \text{ m} \end{aligned}$$

Tower A is 489 m away and Tower B is 575.9 m away.

39. Find the area of the triangle below.



$$A = \frac{1}{2} (15)(15) \sin 102$$

$$A = 110 \text{ mm}^2$$

40. A triangular plaque has side lengths of 8 inches, 13 inches and 15 inches. Use Heron's Formula to find the area of the plaque. ($S = 18$)

$$A = \sqrt{18(18-8)(18-13)(18-15)}$$

$$A = \sqrt{18(10)(5)(3)}$$

$$A = \sqrt{2700}$$

$$A = 52 \text{ in}^2$$

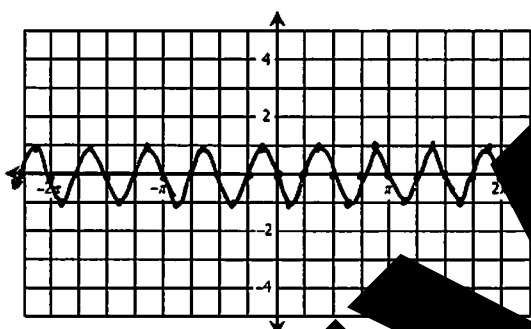
Topic 6: Graphing Trigonometric Functions

Directions: Identify the amplitude and period of each function, then graph.

41. $f(x) = -\sin 4x$

Amplitude: 1

Period: $\pi/2$



42. $f(x) = 2 \sin \frac{\pi}{3} x$

Amplitude: 2

Period: $3\pi/2$



43. $f(x) = \frac{3}{2} \sec \frac{8}{3} x$

Amplitude: undef

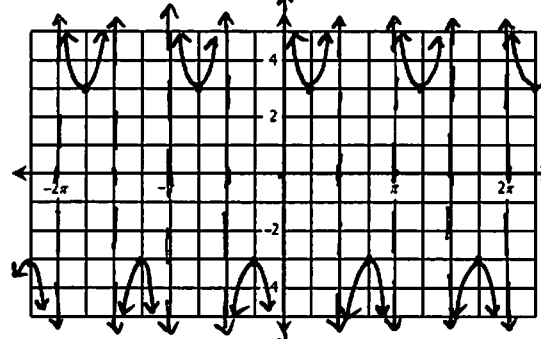
Period: π



44. $f(x) = 3 \cdot \csc 2x$

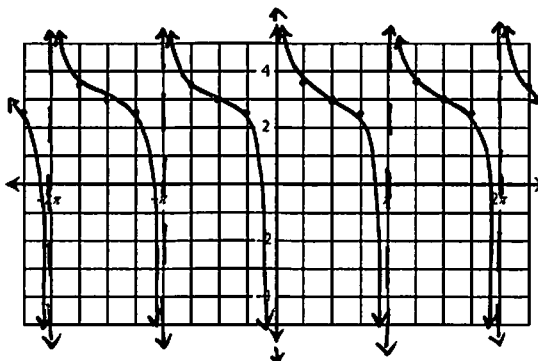
Amplitude: undef

Period: π



Directions: Identify the amplitude, period, translations, and midline of each function. Then graph.

45. $f(x) = \frac{1}{2} \cot(x - \pi) + 3$



Amplitude: undef

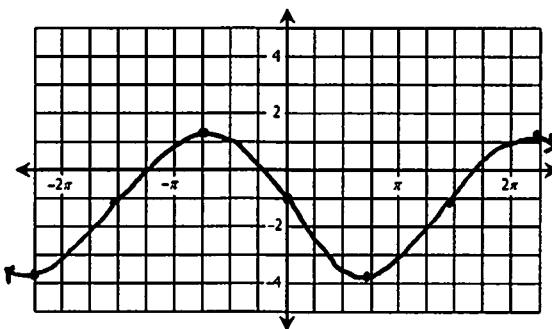
Period: π

Phase Shift: Right π

Vertical Shift: up 3

Midline: $y = 3$

$$46. f(x) = \frac{5}{2} \cdot \cos \frac{2}{3} \left(x + \frac{3\pi}{4} \right) - 1$$



Amplitude: $\frac{5}{2}$

Period: 3π

Phase Shift: $-\frac{3\pi}{4}$

Vertical Shift: -1

Midline: $y = -1$

Topic 7: Inverse Trigonometric Functions & Compositions of Trigonometric Functions

Directions: Give the exact value, if it exists.

47. $\sin^{-1} \frac{\sqrt{2}}{2}$ $\frac{\pi}{4}$

48. $\arctan(-\sqrt{3})$ $-\frac{\pi}{3}$

49. $\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right)$ $\frac{5\pi}{6}$

50. $\cos \left[\cos^{-1} \left(-\frac{2}{7} \right) \right]$ $-\frac{2}{7}$

51. $\tan(\tan^{-1} 3)$ 3

52. $\sin \left[\cos^{-1} \left(\frac{5}{4} \right) \right]$ DNE

53. $\arctan \left[\tan \left(-\frac{\pi}{6} \right) \right]$ $-\frac{\pi}{6}$

54. $\cos^{-1} \left(\cos \frac{2\pi}{3} \right)$ $\frac{2\pi}{3}$

55. $\arcsin \left[\sin \left(-\frac{3\pi}{2} \right) \right]$ $\arcsin \left(\sin \frac{\pi}{2} \right) = \frac{\pi}{2}$

56. $\arccos \left[\cos \left(\frac{11\pi}{6} \right) \right]$ $\arccos \left(\cos \frac{\pi}{6} \right) = \frac{\pi}{6}$

57. $\sin^{-1} \left(\sin \frac{\pi}{3} \right)$ $\sin^{-1} \left(\sin \left(-\frac{\pi}{3} \right) \right) = -\frac{\pi}{3}$

58. $\tan^{-1} \left[\tan \left(-\frac{5\pi}{4} \right) \right]$ $\tan^{-1} \left[\tan \left(-\frac{\pi}{4} \right) \right] = -\frac{\pi}{4}$

59. $\sin \left(\cos^{-1} \left(\frac{\sqrt{2}}{2} \right) \right)$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

60. $\tan[\arcsin(-1)]$ $\tan \left(-\frac{\pi}{2} \right) = \text{DNE}$

61. $\sin \left(\cos^{-1} \left(\frac{1}{2} \right) \right)$ $\sin \left(\frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}$

62. $\arctan(\sin 2\pi)$ $\arctan 0 = 0$

63. $\arccos \left[\sin \left(-\frac{3\pi}{4} \right) \right]$ $\arccos \left(-\frac{\sqrt{2}}{2} \right) = \frac{3\pi}{4}$

64. $\tan^{-1} \left(-\frac{24}{7} \right)$
 $\tan$ in Q2 + Q4
 $\tan^{-1}$ in Q1 + Q4
 $\therefore \theta$ in Q4

Triangle with sides 7, 24, and hypotenuse 25.
 $7^2 + 24^2 = c^2$
 $c = 25$
 $\sin \theta = \frac{-24}{25}$

65. $\tan \left(\cos^{-1} \frac{3\sqrt{5}}{10} \right)$
 $\cos$ in Q1 + Q4
 $\cos^{-1}$ in Q1 + Q2
 $\therefore \theta$ in Q1

Triangle with sides $3\sqrt{5}$, b , and hypotenuse 10.
 $(3\sqrt{5})^2 + b^2 = 10^2$
 $b = \sqrt{55}$
 $\tan \theta = \frac{\sqrt{11}}{3}$

Name: _____

Date: _____ Per: _____

Unit 5 Test

Trigonometric Functions

1. Write 252° in radians.

$$252 \cdot \frac{\pi}{180}$$

$$\frac{7\pi}{5}$$

2. Write $-\frac{2\pi}{9}$ in degrees.

$$-\frac{2\pi}{9} \cdot \frac{180}{\pi}$$

3. When graphed in standard form, the terminal side of an angle measuring $\frac{11\pi}{4}$ lies in which quadrant?

$$495^\circ \rightarrow 135^\circ$$

- A. I
B. II
C. III
D. IV

B

4. Which angles are coterminal? Check all that apply.

☐ $\frac{5\pi}{6}$	☒ $\frac{7\pi}{6}$
☐ $\frac{11\pi}{6}$	☐ $-\frac{17\pi}{6}$

5. Write 61.595° in degree-minute-second form.

$$0.595(60) = 35.7$$

$$0.7(60) = 42$$

$$61^\circ 35' 42''$$

6. Write $30^\circ 27' 15''$ in decimal degrees. Round to the nearest thousandth.

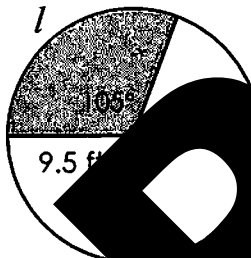
$$27/60 = 0.45$$

$$15/3600 = 0.0042$$

$$230.4542^\circ$$

Use the circle below for questions 7 and 8.

(Round each answer to the nearest tenth.)

7. Find l , the length of the arc of the shaded sector.

$$105^\circ \rightarrow \frac{7\pi}{12}$$

$$9.5 \left(\frac{7\pi}{12} \right)$$

$$l = 17.4 \text{ ft}$$

8. Find A , the area of the shaded sector.

$$A = \frac{1}{2} \left(\frac{7\pi}{12} \right) (9.5)^2$$

$$= \frac{7\pi}{24} (90.25)$$

$$A = 82.7 \text{ ft}^2$$

9. A school bus tire with a diameter of 42 inches is rotating at a rate of 250 revolutions per minute.

Find the linear speed of the bus in miles per hour. Round to the nearest tenth.

$$S = 2\pi(21)(250) = 10500\pi \text{ in} \quad ; \quad V = \frac{10500\pi \text{ in}}{1 \text{ min}}$$

- A. 26.3 mph
B. 26.5 mph
C. 28.8 mph
D. 31.2 mph

$$\frac{10500\pi \text{ in}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 31.2 \text{ mph}$$

D

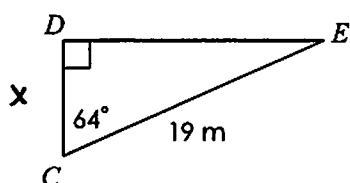
10. Find the angular speed of the tire in radians per minute. Round to the nearest tenth.

$$\theta = 2\pi(250) = 500\pi \text{ radians}$$

$$W = \frac{500\pi \text{ rad}}{1 \text{ min}} \approx 1570.8 \text{ rad/min}$$

- A. 1,570.8 radians/minute
- B. 942.5 radians/minute
- C. 1,292.6 radians/minute
- D. 1,804.1 radians/minute

11. Find CD to the nearest tenth of a meter.



$$\cos 64 = \frac{x}{19}$$

$$CD = 8.3 \text{ m}$$

12. Find $m\angle W$ to the nearest degree.



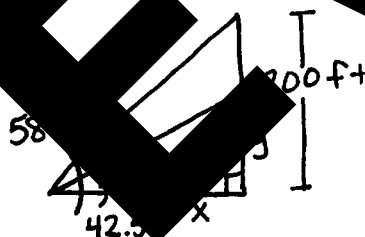
$$\theta = \frac{3}{11}$$

$$\theta = \sin^{-1}\left(\frac{3}{11}\right)$$

$$m\angle W = 15.8^\circ$$

13. A fireman spots a stranded woman standing on a ledge of a 200-foot tall office building. From the point where the fireman is standing, the angle of elevation to the stranded woman is $42^\circ 33'$ and to the top of the building is $58^\circ 6'$. How far from the ground is the stranded woman?

- A. 108.8
- B. 114.3 feet
- C. 124.5 feet
- D. 124.5



$$\tan 58.1 = \frac{200}{x} \rightarrow x = 124.5 \text{ ft}$$

$$\tan 42.55 = \frac{y}{124.5} \rightarrow y = 114.3 \text{ ft}$$

B

14. If $(-1, -1)$ is on the terminal side of θ in standard form, find the exact values of the trigonometric functions of θ .

$$1^2 + 1^2 = r^2$$

$$2 = r^2$$

$$r = \sqrt{2}$$

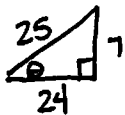
$$x = -1$$

$$y = -1$$

$$r = \sqrt{2}$$

$\sin \theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$	$\csc \theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$
$\cos \theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$	$\sec \theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$
$\tan \theta = \frac{-1}{-1} = 1$	$\cot \theta = \frac{-1}{-1} = 1$

15. If $\cot \theta = -\frac{24}{7}$, and $\cos \theta < 0$, name the quadrant in which θ lies, then find the exact value of $\csc \theta$.



$$\begin{aligned} 7^2 + 24^2 &= c^2 \\ 625 &= c^2 \\ 25 &= c \end{aligned}$$

Quadrant: 2

$$\csc \theta = \frac{25}{7}$$

16. Give the reference angle of $\frac{8\pi}{5}$ in radians.

$$2\pi - \frac{8\pi}{5} = \frac{2\pi}{5}$$

A. $\frac{\pi}{5}$

B. $\frac{2\pi}{5}$

C. $\frac{\pi}{10}$

D. $\frac{3\pi}{5}$

For questions 17-20, give the exact value of each trigonometric function.

17. $\csc \frac{5\pi}{4}$

$$\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

A. $-\frac{2\sqrt{3}}{3}$

C. $-\sqrt{2}$

B. $\frac{2\sqrt{3}}{3}$

D. $\sqrt{2}$

18. $\tan \frac{7\pi}{6}$

A. $\frac{\sqrt{3}}{3}$

B. $-\frac{\sqrt{3}}{3}$

D. $-\sqrt{3}$

A

19. $\cos\left(-\frac{2\pi}{3}\right)$

A. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

B. $-\frac{\sqrt{3}}{2}$

D. $-\frac{1}{2}$

D

20. $\sec\left(-\frac{\pi}{6}\right)$

$$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

A. $-\frac{2\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{3}$

C. $-\frac{\sqrt{3}}{2}$

D. $\frac{\sqrt{3}}{2}$

B

21. Which functions have a value of 1? Check all that apply.



$$\sin\left(-\frac{\pi}{4}\right)$$



$$\cos \pi$$

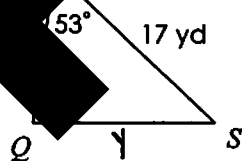


$$\sec \frac{3\pi}{2}$$



$$\cot\left(-\frac{3\pi}{4}\right)$$

22. Solve the triangle below. Round all answers to the nearest tenth.



$$y^2 = 12^2 + 17^2 - 2(12)(17)\cos 53$$

$$y^2 = 433 - 408\cos 53$$

$$y^2 = 187.4595$$

$$y = 13.7$$

$$17^2 = 12^2 + 13.7^2 - 2(12)(13.7)\cos x$$

$$289 = 331.69 - 328.8\cos x$$

$$-42.69 = -328.8\cos x$$

$$.1298 = \cos x$$

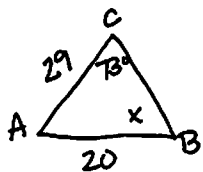
$$x = 82.5^\circ$$

$$m\angle Q = 82.5^\circ$$

$$m\angle S = 44.5^\circ$$

$$QS = 13.7 \text{ yd}$$

23. In $\triangle ABC$, $m\angle C = 73^\circ$, $AB = 20$ km, and $AC = 29$. Find $m\angle B$.



$$\frac{\sin 73}{20} = \frac{\sin x}{29}$$

$$20 \sin x = 29 \sin 73$$

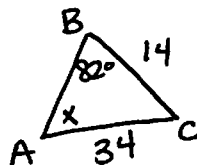
$$\sin x = 1.3866$$

$$x = \emptyset$$

- A. 41.3°
B. 41.3° or 138.7°
C. 65.7°
D. Not possible

D

24. In $\triangle ABC$, $m\angle B = 82^\circ$, $AC = 34$ mm, and $BC = 14$ mm. Find $m\angle A$.



$$\frac{\sin x}{14} = \frac{\sin 82}{34}$$

$$14 \sin 82 = 34 \sin x$$

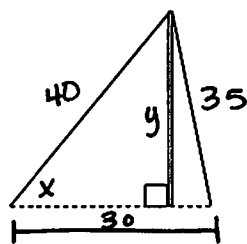
$$\sin x = 0.34$$

$$x = 20.1^\circ$$

- A. 73.9°
B. 41.3° or 155.9°
C. 24.1°
D. Not possible

C

25. Two cables, with lengths of 40 feet and 35 feet, are tied to a pole to stakes in the ground. If the stakes are 30 feet apart, find the height of the pole.



$$35^2 = 30^2 + 40^2 - 2(30)(40) \cos x$$

$$1225 = 2500 - 2400 \cos x$$

$$.53125 = \cos x$$

$$57.9^\circ = x$$

$$\frac{\sin 90}{40} = \frac{\sin 57.9}{y}$$

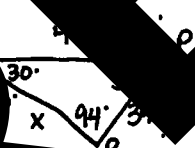
$$y = 33.9$$

- A. 30.6 ft
B. 33.9 ft
C. 36.2 ft
D. 39.5 ft

B

26. Philadelphia, Pennsylvania is 115 miles directly west of Columbus, Ohio. The bearing from Columbus to Richmond, Virginia is $S 60^\circ E$. The bearing from Philadelphia to Richmond is $S 34^\circ W$. Find the distance from Columbus to Richmond to the nearest mile.

- A. 314 miles
B. 321 miles
C. 332 miles
D. 345 miles



$$\frac{\sin 56}{x} = \frac{\sin 94}{115}$$

$$115 \cdot \sin 56 = x \cdot \sin 94$$

$$x = 344.9$$

D

27. Find the area of the triangle below.



$$\frac{\sin 89}{x} = \frac{\sin 40}{11}$$

$$x = 17.1$$

$$A = \frac{1}{2} (11)(17.1) \sin 51$$

$$A = 73.1$$

- A. 73.1 km^2
B. 78.9 km^2
C. 82.4 km^2
D. 85.2 km^2

A

28. A triangular parking lot has sides measuring 90 meters, 54 meters, and 128 meters. Use Heron's Formula to find the area of the parking lot.

$$s = 136$$

$$A = \sqrt{136(136-90)(136-54)(136-128)}$$

$$= \sqrt{136(46)(82)(8)}$$

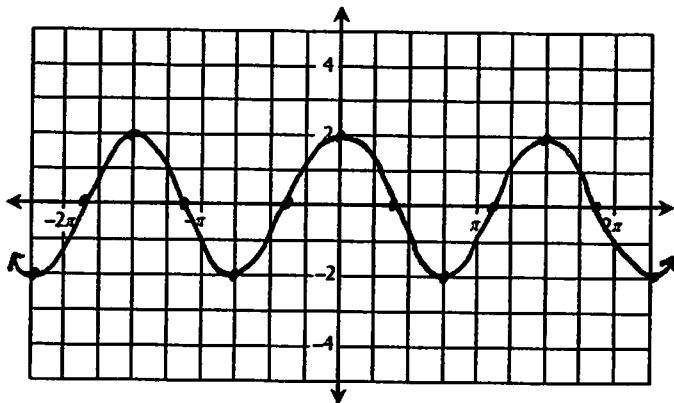
$$= 2025.8$$

- A. $1,805.2 \text{ m}^2$
B. $1,992.7 \text{ m}^2$
C. $2,025.8 \text{ m}^2$
D. $2,308.6 \text{ m}^2$

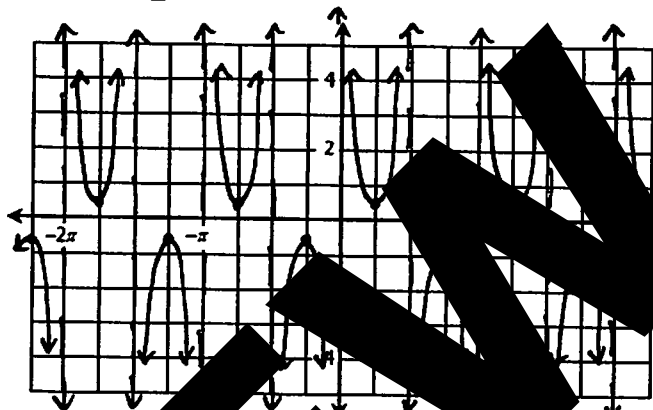
C

Identify the amplitude and period of each function, then graph.

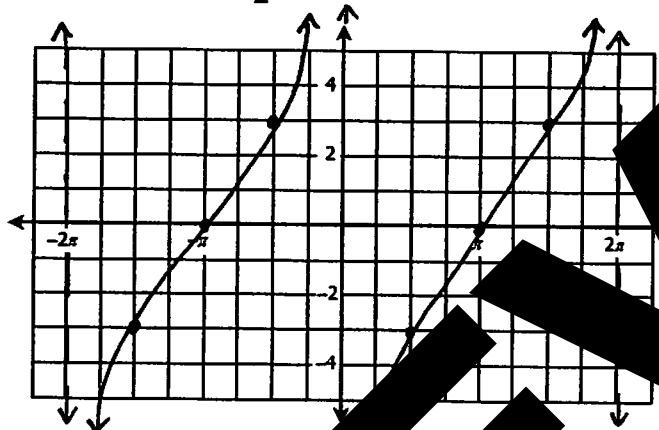
29. $f(x) = 2 \cdot \cos \frac{4}{3}x$ Amp: 2 Per: $\frac{3\pi}{2}$



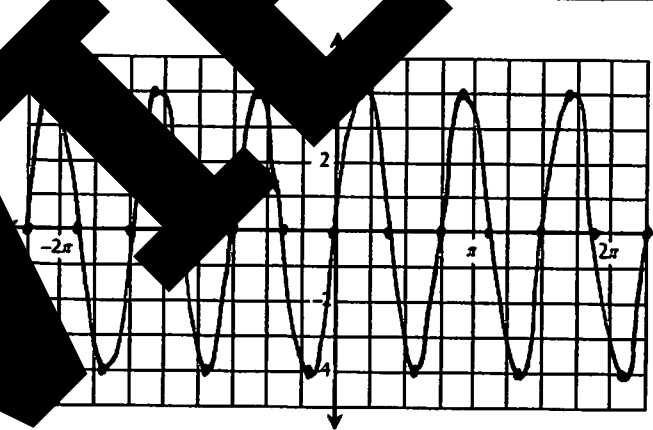
30. $f(x) = \frac{1}{2} \cdot \csc 2x$ Amp: undef Per: π



31. $f(x) = -3 \cdot \cot \frac{1}{2}x$ Amp: undef Per: 2π

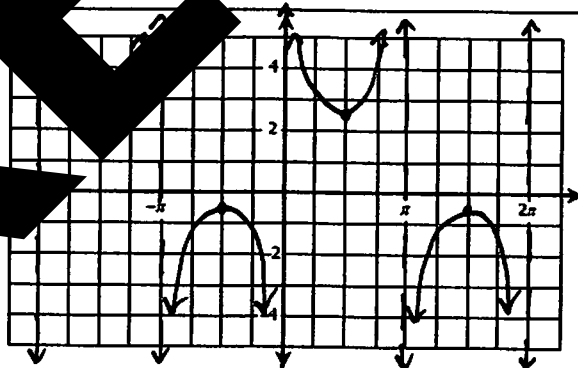


32. $f(x) = \sin \frac{3\pi}{4}x$ Amp: 1 Per: $\frac{3\pi}{4}$



Identify the amplitude, period, phase shifts, and midline of each function. Then graph.

33. $f(x) = \frac{3}{2} \cdot \sec \left(\frac{\pi}{2}x - \pi \right) + 1$



Amplitude: undef

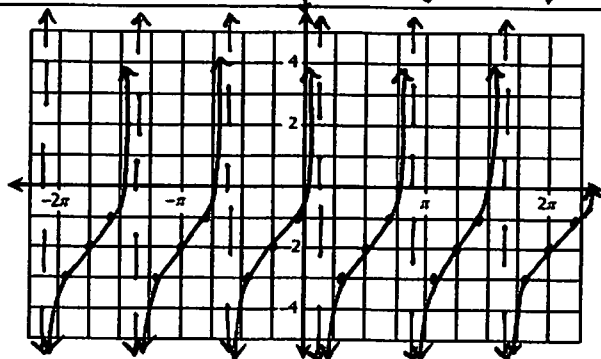
Period: 2π

Phase Shift: Right $\frac{\pi}{2}$

Vertical Shift: Up 1

Midline: $y = 1$

34. $f(x) = \tan \frac{4}{3}(x + \pi) - 2$



Amplitude: undef

Period: $\frac{3\pi}{4}$

Phase Shift: Left π

Vertical Shift: Down 2

Midline: $y = -2$

For questions 35-42, give the exact value, if it exists.

35. $\arccos\left(-\frac{1}{2}\right)$

- A. $-\frac{\pi}{3}$ C. $\frac{2\pi}{3}$
B. $\frac{\pi}{6}$ D. $\frac{5\pi}{6}$

C

36. $\tan^{-1}\frac{\sqrt{3}}{3}$

- A. $\frac{\pi}{6}$ C. $-\frac{\pi}{6}$
B. $\frac{\pi}{3}$ D. $-\frac{\pi}{3}$

A

37. $\tan^{-1}\left(\tan\frac{11\pi}{6}\right)$

$\tan^{-1}\left(\tan^{-\frac{\pi}{6}}\right)$

- A. $-\frac{\pi}{6}$ C. $\frac{\pi}{4}$
B. $-\frac{\pi}{3}$ D. Not possible

A

38. $\cos^{-1}(\tan\frac{\pi}{2})$

- A. 0 C. $\frac{\pi}{2}$
B. π D. Not possible

C

39. $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

- A. $\frac{\pi}{4}$ C. Not possible
B. $-\frac{\pi}{4}$ D. Not possible

B

40. $\sin[\sin^{-1}(-1)]$
 $\tan\left(\frac{9\pi}{2}\right)$

- A. 0 C. -1
B. 1 D. Not possible

D

41. $\cos\left(\frac{\sqrt{3}}{2}\right)$

- A. $-\frac{1}{2}$ C. $-\frac{\sqrt{3}}{2}$
B. $\frac{1}{2}$ D. Not possible

B

42. $\sin\left[\arccos\left(-\frac{\sqrt{7}}{5}\right)\right]$

- A. $\frac{4\sqrt{2}}{5}$ C. $\frac{3\sqrt{2}}{5}$
B. $-\frac{4\sqrt{2}}{5}$ D. $-\frac{3\sqrt{2}}{5}$

C

CREDITS

I use clipart and
fonts in my products



Art with a Penny



Many thanks to these
talented artists!